NMMO401 CONTINUUM MECHANICS, WINTER SEMESTER 2020/2021
SYLLABUS

VÍT PRŮŠA

Topics/notions printed in *italics* are not a part of the exam. All other topics/notions are expected to mastered at the level of a reflex action.

- Preliminaries.
  - Linear algebra.
    - Scalar product, vector product, mixed product, tensor product. Transposed matrix.
    - Tensors. *Inertia tensor as an example of a tensorial quantity in mechanics.*
    - Cofactor matrix $\operatorname{cof} \mathbb{A}$ and determinant $\det \mathbb{A}$. Geometrical interpretation.
    - Cayley–Hamilton theorem, characteristic polynomial, eigenvectors, eigenvalues.
    - Trace of a matrix.
    - Invariants of a matrix and their relation to the eigenvalues and the mixed product.
    - Properties of proper orthogonal matrices, angular velocity.
    - Polar decomposition. Geometrical interpretation.
  - Elementary calculus.
    - Matrix functions. Exponential of a matrix.
    - Representation theorem for scalar valued isotropic tensorial functions and tensor valued isotropic tensorial functions.
    - Gâteaux derivative, Fréchet derivative. Derivatives of the invariants of a matrix.
    - Operators ∇, div and rot for scalar and vector fields. Operators div and rot for tensor fields. Abstract definitions and formulae in Cartesian coordinate system. Identities in tensor calculus.
  - Line, surface and volume integrals.
    - Line integral of a scalar valued function $\int_\gamma \varphi \, \mathrm{d}\boldsymbol{X}$, line integral of a vector valued function $\int_\gamma \boldsymbol{v} \bullet \mathrm{d}\boldsymbol{X}$.
    - Surface integral of a scalar valued function $\int_S \varphi \, \mathrm{dS}$, surface integral of a vector valued function $\int_S \boldsymbol{v} \bullet \mathrm{d}\boldsymbol{S}$, surface Jacobian.
    - Volume integral, Jacobian matrix.
  - Stokes theorem and its consequences.
    - *Potential vector field, path independent integrals, curl free vector fields. Characterisation of potential vector fields.*
  - Elementary concepts in classical physics.
    - Newton laws.
    - Galilean invariance, principle of relativity, non-inertial reference frame.
    - Fictitious forces (Euler, centrifugal, Coriolis).
- Kinematics of continuous medium.
  - Basic concepts.
    - Notion of continuous body. Abstract body, placer, configuration.
    - Reference and current configuration. Lagrangian and Eulerian description.
    - Deformation/motion $\boldsymbol{\chi}$.
    - Local and *global* invertibility of the motion/deformation, condition $\det \mathbb{F} > 0$.
    - Deformation gradient $\mathbb{F}$ and its geometrical interpretation. Polar decomposition $\mathbb{F} = \mathbb{R}\mathbb{U}$ of the deformation gradient and its geometrical interpretation.
    - Relative deformation gradient.
    - *Deformation gradient and polar decompostion for simple shear.*
    - Displacement $\boldsymbol{U}$.
    - Deformation of infinitesimal line, surface a volume elements. Concept of isochoric motion.
    - Lagrangian velocity field $\boldsymbol{V}$, Eulerian velocity field $\boldsymbol{v}$. Material time derivative $\frac{\mathrm{d}}{\mathrm{d}t}$ of Eulerian quantities.
    - Streamlines and pathlines (trajectories).
    - *Relative deformation/motion. Interpretation of $\mathbb{D}$ and $\mathbb{W}$ via time derivatives of relative deformation gradient.*
    - Spatial velocity gradient $\mathbb{L}$, its symmetric part $\mathbb{D}$ and skew-symmetric part $\mathbb{W}$.
  - Strain measures.
    - Left and right Cauchy–Green tensor, $\mathbb{B}$ and $\mathbb{C}$. *Hencky strain.*
    - Green–Saint-Venant strain tensor $\mathbb{E}$, Euler–Almansi strain tensor $\mathbb{e}$. Geometrical interpretation.
    - Linearised strain $\mathbb{e}$.

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  - Compatibility conditions for linearised strain ε in $\mathbb{R}^2$. *Compatibility conditions for linearised strain* ε *in* $\mathbb{R}^3$.
  - Rate quantities.
    * Rate of change of Green–Saint-Venant strain, rate of change of Euler–Almansi strain and their relation to the symmetric part of the velocity gradient $\mathbb{D}$.
    * Rate of change of infinitesimal line, surface and volume elements. Divergence of the Eulerian velocity field and its relation to the change of volume.
    * Objective derivatives of tensorial quantities (Oldroyd derivative).
  - Kinematics of moving surfaces.
    * Lagrange criterion for material surfaces.
  - Reynolds transport theorem.
    * Reynolds transport theorem for the volume moving with the medium.
- Dynamics and thermodynamics of continuous medium.
  - Mechanics.
    * Balance laws for continuous medium as counterparts of the classical laws of Newtonian physics of point particles.
    * Concept of contact/surface forces. *Existence of the Cauchy stress tensor* $\mathbb{T}$ *(tetrahedron argument).*
    * Pure tension, pure compression, tensile stress, shear stress.
    * Balance of mass, linear momentum and angular momentum in Eulerian description.
    * Balance of angular momentum and its implications regarding the symmetry of the Cauchy stress tensor. *Proof of the symmetry of the Cauchy stress tensor.*
    * Balance of mass, linear momentum and angular momentum in Lagrangian description.
    * First Piola–Kirchhoff stress tensor $\mathbb{T}_{\mathrm{R}}$ and its relation to the Cauchy stress tensor $\mathbb{T}$. Piola transformation.
    * Formulation of boundary value problems in Eulerian and Lagrangian descripition, transformation of traction boundary conditions from the current to the reference configuration.
  - Elementary concepts in thermodynamics of continuous medium.
    * Specific internal energy e, energy/heat flux $\boldsymbol{j}_q$.
    * Balance of total energy in the Eulerian and Lagrangian description.
    * Balance of internal energy in the Eulerian and Lagrangian description.
    * Referential heat flux $\boldsymbol{J}_q$.
    * Specific Helmholtz free energy, specific entropy.
  - Boundary conditions.
  - Geometrical linearisation. Incompressibility condition in the linearised setting. Specification of the boundary conditions in the linearised setting.
- Simple constitutive relations.
  - Pressure and thermodynamic pressure, engineering equation of state. Derivation of compressible and incompressible Navier–Stokes fluid model via the representation theorem for tensor valued isotropic tensorial functions. Complete thermodynamical description of a compressible viscous heat conducting fluid – Navier–Stokes–Fourier equations.
  - Cauchy elastic material. Derivation via the representation theorem for tensor valued isotropic tensorial functions.
  - Green elastic material. (Hyperelastic solid.) Relation between the specific Helmholtz free energy and the Cauchy stress tensor for an elastic solid.
  - *Gough–Joule effect.*
- *Simple problems in the mechanics of continuous medium.*
  - *Archimedes law.*
  - *Deformation of a cylinder (linearised elasticity). Hooke law.*
  - *Inflation of a hollow cylinder made of an incompressible isotropic elastic solid. (Comparison of the linearised elasticity theory and fully nonlinear theory.)*
  - *Waves in the linearised isotropic elastic solid.*
  - *Stability of the rest state of the incompressible Navier–Stokes fluid.*
  - *Drag acting on a rigid body moving with a uniform velocity in the incompressible Navier–Stokes fluid.*

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