1. Consider linearised homogeneous isotropic elastic solid, that is a continuous medium where the stress tensor is given by the formula
$$\mathbb{T} = \lambda \left(\operatorname{Tr} \mathbb{\epsilon}\right) \mathbb{I} + 2\mu \mathbb{\epsilon},$$
where $\mathbb{\epsilon} =_{\text{def}} \frac{1}{2}\left(\nabla \boldsymbol{U} + (\nabla \boldsymbol{U})^{\top}\right)$ is the linearised strain. Show that the (linearised) governing equations in $\mathbb{R}^3$, that is
$$\rho \frac{\partial^2 \boldsymbol{U}}{\partial t^2} = \operatorname{div} \mathbb{T} + \rho \boldsymbol{b},$$
admit, if there are no specific body forces, $\boldsymbol{b} = \boldsymbol{0}$, a solution in the form of a wave
$$\boldsymbol{U} = \boldsymbol{A} \sin\left(\boldsymbol{K} \bullet \boldsymbol{X} - \omega t\right),$$
where $\boldsymbol{A}$ denotes the amplitude of the wave, vector $\boldsymbol{K}$ denotes the wave vector that determines the direction of the propagation of the wave and the spatial frequency of the wave, and ω is the angular frequency. (The speed of propagation of the wave is given by the formula $c =_{\text{def}} \frac{\omega}{K}$, where $K =_{\text{def}} |\boldsymbol{K}|$ is called the wavenumber.)
In particular, show that $\boldsymbol{A} \sin\left(\boldsymbol{K} \bullet \boldsymbol{X} - \omega t\right)$ is a solution to the (linearised) governing equations provided that *either* $\boldsymbol{A}$ is parallel to $\boldsymbol{K}$ and the speed of propagation is $c_{\parallel} = \sqrt{\frac{\lambda + 2\mu}{\rho}}$ *or* $\boldsymbol{A}$ is perpendicular to $\boldsymbol{K}$ and the speed of propagation is $c_{\perp} = \sqrt{\frac{\mu}{\rho}}$.

Having solved the problem, you can enjoy reading the following paper on seismic waves, *František Gallovič: Pár sekund k dobru: Systémy včasného varování před zemětřeseními (in Czech), Vesmír 91, 510-512, 2012.*