1. Use Stokes theorem and prove the following lemma. Let u, v are smooth scalar valued functions, $v : \mathbb{R}^3 \mapsto \mathbb{R}$, $u : \mathbb{R}^3 \mapsto \mathbb{R}$, $\boldsymbol{v}$ a smooth vector field, $\boldsymbol{v} : \mathbb{R}^3 \mapsto \mathbb{R}^3$, and let $\mathbb{A}$ be a smooth tensor valued function $\mathbb{A} : \mathbb{R}^3 \mapsto \mathbb{R}^{3\times 3}$. Let $\Omega \subset \mathbb{R}^3$ be a bounded domain with smooth boundary, then

$$\int_{\Omega} u \left(\nabla v\right) \, \mathrm{dv} = \int_{\partial\Omega} uv \, \mathrm{d}\boldsymbol{S} - \int_{\Omega} \left(\nabla u\right) v \, \mathrm{dv},$$

$$\int_{\Omega} \left(\operatorname{div} \mathbb{A}\right) \bullet \boldsymbol{v} \, \mathrm{dv} = \int_{\partial\Omega} \left(\mathbb{A}^{\top} \boldsymbol{v}\right) \bullet \mathrm{d}\boldsymbol{S} - \int_{\Omega} \mathbb{A} : \nabla \boldsymbol{v} \, \mathrm{dv}.$$