1. Let φ, ψ, $\boldsymbol{u}$, $\boldsymbol{v}$ and $\mathbb{A}$ be smooth scalar, vector and tensor fields in $\mathbb{R}^3$. Show that[1]

$$
\begin{aligned}
\operatorname{div}(\varphi \boldsymbol{v}) &= \boldsymbol{v} \bullet (\nabla \varphi) + \varphi \operatorname{div} \boldsymbol{v}, \\
\operatorname{div}(\boldsymbol{u} \times \boldsymbol{v}) &= \boldsymbol{v} \bullet \operatorname{rot} \boldsymbol{u} - \boldsymbol{u} \bullet \operatorname{rot} \boldsymbol{v}, \\
\operatorname{div}(\boldsymbol{u} \otimes \boldsymbol{v}) &= [\nabla \boldsymbol{u}] \boldsymbol{v} + \boldsymbol{u} \operatorname{div} \boldsymbol{v}, \\
\operatorname{div}(\varphi \mathbb{A}) &= \mathbb{A} (\nabla \varphi) + \varphi \operatorname{div} \mathbb{A}.
\end{aligned}
$$

Further, show that

$$
\begin{aligned}
\nabla (\varphi \psi) &= \psi \nabla \varphi + \varphi \nabla \psi, \\
\nabla (\varphi \boldsymbol{v}) &= \boldsymbol{v} \otimes \nabla \varphi + \varphi \nabla \boldsymbol{v}, \\
\nabla (\boldsymbol{u} \bullet \boldsymbol{v}) &= (\nabla \boldsymbol{u})^{\top} \boldsymbol{v} + (\nabla \boldsymbol{v})^{\top} \boldsymbol{u}, \\
\operatorname{rot}(\varphi \boldsymbol{v}) &= \varphi \operatorname{rot} \boldsymbol{v} - \boldsymbol{v} \times \nabla \varphi.
\end{aligned}
$$

[1] We use the notation $([\nabla \boldsymbol{u}] \boldsymbol{v})_i = \frac{\partial \mathrm{u}_i}{\partial x_j} \mathrm{v}_j$.