1. Let $\boldsymbol{a} \in \mathbb{R}^3$ be a unit vector, and let $\boldsymbol{u} \in \mathbb{R}^3$ be an arbitrary vector. Show that vectors $\boldsymbol{a} \times \boldsymbol{u}$ and $(\mathbb{I} - \boldsymbol{a} \otimes \boldsymbol{a})\,\boldsymbol{u}$ have the same length.

2. Let $\mathbb{A} \in \mathbb{R}^{3\times 3}$ a $\mathbb{B} \in \mathbb{R}^{3\times 3}$ be invertible matrices. Show that

$$\det(\mathbb{A} + \mathbb{B}) = \det \mathbb{A} + \operatorname{Tr}\left(\mathbb{A}^{\top} \operatorname{cof} \mathbb{B}\right) + \operatorname{Tr}\left(\mathbb{B}^{\top} \operatorname{cof} \mathbb{A}\right) + \det \mathbb{B},$$

   where $\operatorname{cof} \mathbb{C} =_{\text{def}} (\det \mathbb{C})\, \mathbb{C}^{-\top}$ denotes the cofactor matrix of matrix $\mathbb{C}$.

3. [Optional] We have defined the norm of a matrix $\mathbb{A}$ as

$$|\mathbb{A}| =_{\text{def}} \left(\operatorname{Tr}\left(\mathbb{A}\mathbb{A}^{\top}\right)\right)^{\frac{1}{2}}.$$

   If one interprets the matrix as a linear mapping, there is another possibility to define a norm, namely

$$|\mathbb{A}|_{\text{op}} =_{\text{def}} \sup_{\boldsymbol{x} \in \mathbb{R}^3, \boldsymbol{x} \neq \boldsymbol{0}} \frac{|\mathbb{A}\boldsymbol{x}|_{\mathbb{R}^3}}{|\boldsymbol{x}|_{\mathbb{R}^3}}.$$

   (This is the way how the norm is defined in functional analysis, $|\cdot|_{\mathbb{R}^3}$ denotes the standard Euclidean norm.) Is the norm $|\mathbb{A}|_{\text{op}}$ the same norm as $|\mathbb{A}|$? If not, show that it is an equivalent norm. (Recall that two norms $|\cdot|_{\text{A}}$ and $|\cdot|_{\text{B}}$ on space X are equivalent if there exist positive constants c_1 and c_2 such that $c_1 |\boldsymbol{x}|_{\text{A}} \leq |\boldsymbol{x}|_{\text{B}} \leq c_2 |\boldsymbol{x}|_{\text{A}}$ holds for all $\boldsymbol{x} \in X$.)

Please check your notes from previous lectures such as mathematical analysis, and be ready for the discussion of the notions such as the line integral of a scalar/vector field, the surface integral of a scalar/vector field, potential of a vector field, Stokes theorem, operators div and rot. I need to know exactly what are you already familiar with, and what we need to carefully define/discuss in the lecture.

For those who are interested in additional reading: Detailed discussion of representation theorems for tensorial functions can be found in a review paper by Zheng (1994). Detailed discussion of the relation between proper orthogonal matrices and rotations can be found in (Ciarlet, 1988, Theorem 1.8-1).

Ciarlet, P. G. (1988). *Mathematical elasticity. Vol. I*, Volume 20 of *Studies in Mathematics and its Applications*. Amsterdam: North-Holland Publishing Co. Three-dimensional elasticity.

Zheng, Q.-S. (1994). Theory of representations for tensor functions – A unified invariant approach to constitutive equations. *Applied Mechanics Reviews 47*(11), 545–587.