CHAPTER 1

BEAM-COLUMNS

1.1. Introduction. In the elementary theory of bending, it is found that stresses and deflections in beams are directly proportional to the applied loads. This condition requires that the change in shape of the beam due to bending must not affect the action of the applied loads. For example, if the beam in Fig. 1-1*a* is subjected to only lateral loads, such as Q_1 and Q_2, the presence of the small deflections δ_1 and δ_2 and slight changes in the vertical lines of action of the loads will have only an insignificant effect on the moments and shear forces. Thus it is possible to make calculations for deflections, stresses, moments, etc., on the basis of the initial configuration of the beam. Under these conditions, and also if Hooke's law holds for the material, the deflections are proportional to the acting forces and the principle of superposition is valid; i.e., the final deformation is obtained by summation of the deformations produced by the individual forces.

Conditions are entirely different when both axial and lateral loads act simultaneously on the beam (Fig. 1-1*b*). The bending moments, shear forces, stresses, and deflections in the beam will not be proportional to the magnitude of the axial load. Furthermore, their values will be dependent upon the magnitude of the deflections produced and will be sensitive to even slight eccentricities in the application of the axial load. Beams subjected to axial compression and simultaneously supporting lateral loads are known as *beam-columns*. In this first chapter, beam-columns of symmetrical cross section and with various conditions of support and loading will be analyzed.[1]

1.2. Differential Equations for Beam-columns. The basic equations for the analysis of beam-columns can be derived by considering the beam in Fig. 1-2*a*. The beam is subjected to an axial compressive force P and to a distributed lateral load of intensity q which varies with the distance x along the beam. An element of length dx between two cross sections taken normal to the original (undeflected) axis of the beam is

[1] For an analysis of beams subjected to axial tension see Timoshenko, "Strength of Materials," 3d ed., part II, p. 41, D. Van Nostrand Company, Inc., Princeton, N.J., 1956.

shown in Fig. 1-2*b*. The lateral load may be considered as having constant intensity q over the distance dx and will be assumed positive when in the direction of the positive y axis, which is downward in this case. The shearing force V and bending moment M acting on the sides of the element are assumed positive in the directions shown.

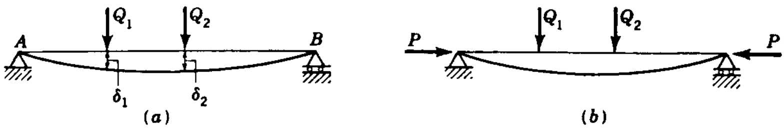

FIG. 1-1

The relations among load, shearing force V, and bending moment are obtained from the equilibrium of the element in Fig. 1-2*b*. Summing forces in the y direction gives

$$-V + q\,dx + (V + dV) = 0$$

or

$$q = -\frac{dV}{dx} \tag{1-1}$$

Taking moments about point n and assuming that the angle between the axis of the beam and the horizontal is small, we obtain

$$M + q\,dx\,\frac{dx}{2} + (V + dV)\,dx - (M + dM) + P\,\frac{dy}{dx}\,dx = 0$$

If terms of second order are neglected, this equation becomes

$$V = \frac{dM}{dx} - P\,\frac{dy}{dx} \tag{1-2}$$

If the effects of shearing deformations and shortening of the beam axis are neglected, the expression for the curvature of the axis of the beam is

$$EI\,\frac{d^2y}{dx^2} = -M \tag{1-3}$$

The quantity EI represents the flexural rigidity of the beam in the plane of bending, that is, in the xy plane, which is assumed to be a plane of symmetry. Combining Eq. (1-3) with Eqs. (1-1) and (1-2), we can express the differential equation of the axis of the beam in the following alternate forms:

$$EI\,\frac{d^3y}{dx^3} + P\,\frac{dy}{dx} = -V \tag{1-4}$$

and

$$EI\,\frac{d^4y}{dx^4} + P\,\frac{d^2y}{dx^2} = q \tag{1-5}$$

Equations (1-1) to (1-5) are the basic differential equations for bending of beam-columns. If the axial force P equals zero, these equations reduce to the usual equations for bending by lateral loads only.

Instead of taking an element dx with sides perpendicular to the x axis (Fig. 1-2b), we can cut an element with sides normal to the deflected axis of the beam (Fig. 1-2c). Since the slope of the beam is small, the normal forces acting on the sides of the element can be taken equal to the axial compressive force P. The shearing force N in this case is related to the shearing force V in Fig. 1-2b by the expression

$$N = V + P\frac{dy}{dx} \tag{a}$$

and instead of Eqs. (1-1) and (1-2) we obtain

$$q = -\frac{dN}{dx} + P\frac{d^2y}{dx^2} \tag{1-1a}$$

and

$$N = \frac{dM}{dx} \tag{1-2a}$$

Equations (1-1a) and (1-2a) can also be derived by considering the equilibrium of the element in Fig. 1-2c. Finally, combining Eq. (1-2a) with Eq. (1-3) yields the equation

$$EI\frac{d^3y}{dx^3} = -N \tag{1-4a}$$

Equation (1-5) remains valid for the element in Fig. 1-2c. Thus we have two sets of differential equations for a beam-column, depending on whether the shearing force is taken on a cross section normal to the deflected or the undeflected axis of the beam.

1.3. Beam-column with a Concentrated Lateral Load. As the first example of the use of the beam-column equations, let us consider a beam of length l on two simple supports (Fig. 1-3) and carrying a single lateral

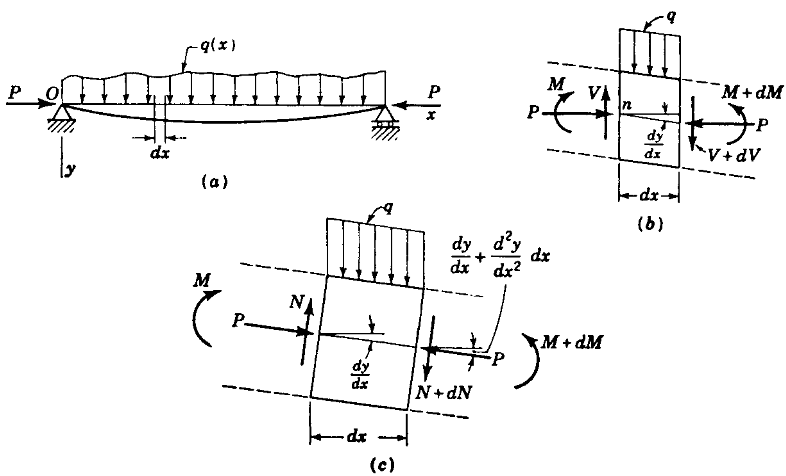

FIG. 1-2