NMMO401 CONTINUUM MECHANICS, WINTER SEMESTER 2016/2017
SYLLABUS

VÍT PRŮŠA

Topics/notions printed in *italics* are not a part of the exam. All other topics/notions are expected to mastered at the level of a reflex action.

- Preliminaries.
  - Linear algebra.
    * Scalar product, vector product, mixed product, tensor product. Transposed matrix.
    * Tensors. *Inertia tensor as an example of a tensorial quantity in mechanics.*
    * Cofactor matrix $\operatorname{cof}\mathbb{A}$ and determinant $\det\mathbb{A}$. Geometrical interpretation.
    * Cayley–Hamilton theorem, characteristic polynomial, eigenvectors, eigenvalues.
    * Trace of a matrix.
    * Invariants of a matrix and their relation to the eigenvalues and the mixed product.
    * Properties of proper orthogonal matrices, angular velocity.
    * Polar decomposition. Geometrical interpretation.
    * Spectral decomposition.
  - Elementary calculus.
    * Matrix functions. Exponential of a matrix. *Skew-symmetric matrices as infinitesimal generators of the group of rotations.*
    * Representation theorem for scalar valued isotropic tensorial functions and tensor valued isotropic tensorial functions.
    * Gâteaux derivative, Fréchet derivative. Derivatives of the invariants of a matrix.
    * Operators ∇, div and rot for scalar and vector fields. Operators div and rot for tensor fields. Abstract definitions and formulae in Cartesian coordinate system. Identities in tensor calculus.
  - Line, surface and volume integrals.
    * Line integral of a scalar valued function $\int_\gamma \varphi \,\mathrm{d}\boldsymbol{X}$, line integral of a vector valued function $\int_\gamma \boldsymbol{v} \bullet \mathrm{d}\boldsymbol{X}$.
    * Surface integral of a scalar valued function $\int_S \varphi \,\mathrm{dS}$, surface integral of a vector valued function $\int_S \boldsymbol{v} \bullet \mathrm{d}\boldsymbol{S}$, surface Jacobian.
    * Volume integral, Jacobian matrix.
  - Stokes theorem and its consequences.
    * *Green identities.*
    * *Potential vector field, path independent integrals, curl free vector fields. Characterisation of potential vector fields.*
    * *Helmholtz decomposition,* $\boldsymbol{v} = -\nabla\varphi + \operatorname{rot}\mathbb{A}$.
    * *Korn equality.*
  - Elementary concepts in classical physics.
    * Newton laws.
- Kinematics of continuous medium.
  - Basic concepts.
    * Notion of continuous body. Abstract body, placer, configuration.
    * Reference and current configuration. Lagrangian and Eulerian description.
    * Deformation/motion $\boldsymbol{\chi}$.
    * Local and *global* invertibility of the motion/deformation.
    * Deformation gradient $\mathbb{F}$ and its geometrical interpretation. Polar decomposition of the deformation gradient and its geometrical interpretation.
    * *Deformation gradient and polar decompostion for simple shear.*
    * Displacement $\boldsymbol{U}$.
    * Deformation of infinitesimal line, surface a volume elements. Concept of isochoric motion.
    * Lagrangian velocity field $\boldsymbol{V}$, Eulerian velocity field $\boldsymbol{v}$. Material time derivative $\frac{\mathrm{d}}{\mathrm{d}t}$ of Eulerian quantities.
    * Streamlines and pathlines (trajectories).
    * *Relative deformation/motion. Interpretation of* $\mathbb{D}$ *and* $\mathbb{W}$ *via time derivatives of relative deformation gradient.*
    * Spatial velocity gradient $\mathbb{L}$, its symmetric $\mathbb{D}$ and skew-symmetric part $\mathbb{W}$.
  - Strain measures.
    * Left and right Cauchy–Green tensor, $\mathbb{B}$ and $\mathbb{C}$.

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    - Green–Saint-Venant strain tensor $\mathbb{E}$, Euler–Almansi strain tensor $\mathbb{e}$. Geometrical interpretation.
    - Linearised strain ε.
  - Compatibility conditions for linearised strain ε in $\mathbb{R}^2$. *Compatibility conditions for linearised strain* ε *in* $\mathbb{R}^3$.
  - Rate quantities.
    - Rate of change of Green–Saint-Venant strain, rate of change of Euler–Almansi strain and their relation to the symmetric part of the velocity gradient $\mathbb{D}$.
    - Rate of change of infinitesimal line, surface and volume elements. Divergence of Eulerian velocity field and its relation to the change of volume.
  - Kinematics of moving surfaces.
    - Lagrange criterion for material surfaces.
  - Reynolds transport theorem.
- Dynamics of continuous medium.
  - Mechanics.
    - Balance laws for continuous medium as counterparts of the classical laws of Newtonian physics of point particles.
    - Concept of contact/surface forces. *Existence of the Cauchy stress tensor* $\mathbb{T}$ *(tetrahedron argument).*
    - Pure tension, pure compression, tensile stress, shear stress.
    - Balance of mass, linear momentum and angular momentum in Eulerian description.
    - Balance of angular momentum and its implications with respect to the symmetry of the Cauchy stress tensor. *Proof of the symmetry of the Cauchy stress tensor.*
    - Balance of mass, linear momentum and angular momentum in Lagrangian description.
    - First Piola–Kirchhoff tensor $\mathbb{T}_{\mathrm{R}}$. Piola transformation.
    - Formulation of boundary value problems in Eulerian and Lagrangian descripition, transformation of traction boundary conditions from the current to the reference configuration.
  - Elementary concepts in thermodynamics of continuous medium.
    - Internal energy, heat flux.
    - Balance of total energy in Eulerian and Lagrangian description.
    - Balance of internal energy in Eulerian and Lagrangian description.
    - Referential heat flux.
  - Boundary conditions.
  - Geometrical linearisation. Incompressibility condition in the linearised setting. Specification of the boundary conditions in the linearised setting.
- Simple constitutive relations.
  - Pressure and thermodynamic pressure. Derivation of compressible and incompressible Navier–Stokes fluid model via the representation theorem for tensor valued isotropic tensorial functions.
  - Cauchy elastic material. Derivation via the representation theorem for tensor valued isotropic tensorial functions.
  - Physical units, dimensionless quantities, Reynolds number.
- *Simple problems in mechanics of continuous medium.*
  - *Archimedes law.*
  - *Deformation of a cylinder (linearised elasticity). Hooke law.*
  - *Inflation of a hollow cylinder made of an incompressible isotropic elastic solid. (Comparison of linearised theory and fully nonlinear theory.)*
  - *Isothermal atmosphere versus swimming pool. (Difference between the notion of the pressure in the case of compressible and incompressible Navier–Stokes fluid model.)*
  - *Stokes formula via dimensional analysis.*
  - *Waves in linearised isotropic elastic solid.*

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