

1. Consider a linearised homogeneous isotropic elastic solid, that is a continuous medium where the Cauchy stress tensor is given by the formula

$$\mathbb{T} = \lambda (\text{Tr } \varepsilon) \mathbb{1} + 2\mu \varepsilon,$$

where $\varepsilon =_{\text{def}} \frac{1}{2} (\nabla \mathbf{U} + (\nabla \mathbf{U})^T)$ is the linearised strain. Show that the (linearised) governing equations

$$\text{div } \mathbb{T} = \mathbf{0},$$

$$\mathbb{T} \mathbf{n} = \mathbf{f},$$

for a steady state in \mathbb{R}^2 admit, if there are no specific body forces, a solution in the form

$$\mathbb{T} = \begin{bmatrix} \frac{\partial^2 \psi}{\partial x_2^2} & -\frac{\partial^2 \psi}{\partial x_1 \partial x_2} \\ -\frac{\partial^2 \psi}{\partial x_1 \partial x_2} & \frac{\partial^2 \psi}{\partial x_1^2} \end{bmatrix},$$

where ψ is called the Airy stress function. Using the *compatibility conditions* in \mathbb{R}^2 show that the Airy stress function can not be just *any function* but it must be a solution to the biharmonic equation

$$\Delta \Delta \psi = 0,$$

with suitable boundary conditions. (This means that the problems in plane elasticity can be reduced to a single scalar equation.)

2. If you want to have some fun with calculations based purely on the dimensional considerations, you can read a short note discussing the estimate of the energy released in the first atomic bomb explosion. (A nice Christmas topic, isn't it?) The note summarises a part of the famous papers The formation of a blast wave by a very intense explosion: I. Theoretical discussion and The formation of a blast wave by a very intense explosion: II. The atomic explosion of 1945 by Geoffrey Taylor.