1. Let $\mathbb{T}_{\mathrm{R}}$ and $\mathbb{T}$ denote the first Piola–Kirchhoff tensor and the Cauchy stress tensor respectively. Show that

$$\operatorname{Div} \mathbb{T}_{\mathrm{R}} = (\det \mathbb{F}) \operatorname{div} \mathbb{T},$$

or, in detail, that

$$\operatorname{Div}_{\mathbf{X}} \mathbb{T}_{\mathrm{R}}(\mathbf{X}, T) = (\det \mathbb{F}(\mathbf{X}, t)) \left. (\operatorname{div}_{\mathbf{x}} \mathbb{T}(\mathbf{x}, t))\right|_{\mathbf{x}=\boldsymbol{\chi}(\mathbf{X},t)},$$

where $\mathbb{F}$ denotes the deformation gradient and $\boldsymbol{\chi}(\mathbf{X}, t)$ is the deformation. (Direct differentiation is not a good idea.)