

Příklady na 3. týden

## Limity funkcí I

1. Dokažte z definice, že

$$\text{a) } \lim_{x \rightarrow 1} \left(\frac{x}{2}\right)^3 = \frac{1}{8} \quad \text{b) } \lim_{x \rightarrow 1^+} [x] = 1 \quad \text{c) } \lim_{x \rightarrow 1^-} [x] = 0$$

Spočítejte

$$2. \text{ (a) } \lim_{x \rightarrow 0} \frac{x^2 - 1}{2x^2 - x - 1} \quad \text{(b) } \lim_{x \rightarrow 1} \frac{x^2 - 1}{2x^2 - x - 1}$$

$$3. \lim_{x \rightarrow 2} \left( \frac{1}{x^2 - 2x} - \frac{x}{x^2 - 4} \right)$$

$$4. \lim_{x \rightarrow 0} \frac{(1+x)(1+2x)\dots(1+nx) - 1}{x}$$

$$5. \lim_{x \rightarrow 1} \frac{x^{100} - 2x + 1}{x^{50} - 2x + 1}$$

$$6. \lim_{x \rightarrow 0} \frac{(1+mx)^n - (1+nx)^m}{x^2}$$

$$7. \lim_{x \rightarrow 1} \frac{x^{n+1} - (n+1)x + n}{(x-1)^2}$$

$$8. \lim_{x \rightarrow 1} \frac{x + x^2 + \dots + x^n - n}{x - 1}$$

$$9. \lim_{x \rightarrow 1} \left( \frac{m}{1-x^m} - \frac{n}{1-x^n} \right)$$

$$10. \lim_{x \rightarrow 0} \frac{\frac{2}{x^2} + 1}{\sqrt{\frac{3}{x^4} - \frac{6}{x^2} + 5}}$$

$$11. \lim_{x \rightarrow 0^+} \frac{(\sqrt{\frac{1}{x^2} + 1} - \sqrt{\frac{1}{x^2} - 1})}{x}$$

$$12. \lim_{x \rightarrow 0^+} \left( \sqrt{\frac{1}{x} + \sqrt{\frac{1}{x} + \sqrt{\frac{1}{x}}}} - \sqrt{\frac{1}{x} - \sqrt{\frac{1}{x} + \sqrt{\frac{1}{x}}}} \right)$$

$$13. \text{ (a) } \lim_{x \rightarrow 16} \frac{\sqrt[4]{x} - 2}{\sqrt{x} - 4} \quad \text{(b) } \lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x}$$

$$14. \lim_{x \rightarrow 0} \frac{\sqrt{1-2x-x^2} - (1-x)}{x}$$

$$15. \lim_{x \rightarrow 0} \frac{\sqrt[3]{27+x} - \sqrt[3]{27-x}}{x + 2\sqrt[3]{x^4}}$$

$$16. \lim_{x \rightarrow 0} \frac{\sqrt[m]{1+x} - \sqrt[n]{1+x}}{x}$$

$$17. \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt[3]{1-x}}{\sqrt[3]{1+x} - \sqrt{1-x}}$$

$$18. \lim_{x \rightarrow a^+} \frac{\sqrt{x} - \sqrt{a} + \sqrt{x-a}}{\sqrt{x^2 - a^2}}, a \geq 0$$

$$19. \lim_{x \rightarrow 0} \frac{\sqrt[m]{1+ax} \sqrt[n]{1+bx} - 1}{x}$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$20. \lim_{x \rightarrow a} \frac{\operatorname{tg} x - \operatorname{tg} a}{x - a}, a \in R$$

$$21. \lim_{x \rightarrow 0} \frac{\sqrt{1 - \cos x^2}}{1 - \cos x}$$

$$22. \lim_{x \rightarrow 0} \frac{\operatorname{tg} x - \sin x}{x^3}$$

$$23. \lim_{x \rightarrow 0} \frac{1 - \cos x \cos 2x \cos 3x}{1 - \cos x}$$

$$24. \lim_{x \rightarrow \pi} \frac{\sin nx}{\sin mx}, n, m \in N$$

$$25. \lim_{x \rightarrow 1} \frac{\sin \pi x}{1 - x}$$

$$26. \lim_{x \rightarrow \frac{\pi}{4}} (\operatorname{tg} 2x) \operatorname{tg} \left( \frac{\pi}{4} - x \right)$$

$$27. \lim_{x \rightarrow 0} \frac{\sin(a+2x) - 2\sin(a+x) + \sin a}{x^2}, a \in R$$

$$28. \lim_{x \rightarrow 0} \frac{\operatorname{cotg}(a+2x) - 2\operatorname{cotg}(a+x) + \operatorname{cotg} a}{x^2}, \sin a \neq 0$$