Modelling of Phase Transformations in magnetostrictive materials like NiMnGa

Tomáš Roubíček

Charles University, Prague
&
Academy of Sciences of the Czech Rep.
&
University of West Bohemia

reflecting collaboration with

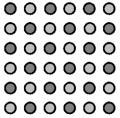
Giuseppe Tomassetti
and
M.Arndt, M.Griebel, V.Novák, P.Plecháč, P.Podio-Guidugli, K.R.Rajagopal, P.Šittner, C.Zanini and others.

Content of the talk:

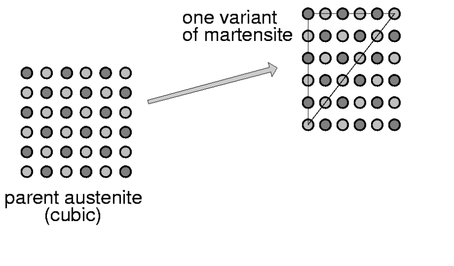
1. Phase transformations in NiMnGa
   - Martensitic/austenitic transformation
   - Ferro/para-magnetic transformation
   - Coupling of transformations: magnetostriction
2. The model and its analysis
   - Partly linearized ansatz
   - Analysis: semi-implicit discretisation, a-priori estimates
   - Analysis: convergence
3. Some other phenomena to be involved
   - General nonlinear ansatz
   - Pinning effects

Shape-memory materials (SMM): alloys (=SMAs) or intermetalics.
The mechanism behind shape-memory effect (=SME):

◦ higher temperatures:
atoms tend to form a latice with high symmetry (mostly cubic):
austenite phase, higher heat capacity

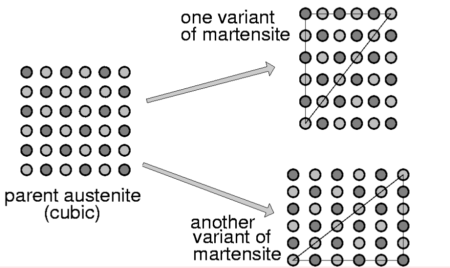


Shape-memory materials (SMM): alloys (=SMAs) or intermetalics.
The mechanism behind shape-memory effect (=SME):
∘ higher temperatures:
  atoms tend to form a latice with high symmetry (mostly cubic):
  austenite phase, higher heat capacity
  a lower-symmetrical latice: martensite phase, lower heat capacity.



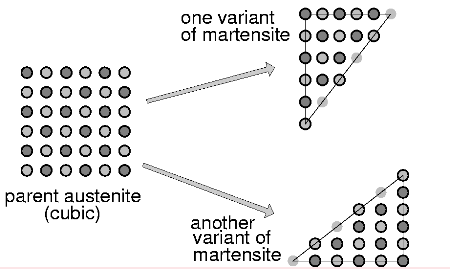
Shape-memory materials (SMM): alloys (=SMAs) or intermetalics.
The mechanism behind shape-memory effect (=SME):

◦ higher temperatures:
atoms tend to form a latice with high symmetry (mostly cubic):
austenite phase, higher heat capacity
a lower-symmetrical latice: martensite phase, lower heat capacity.
the lower-symmetrical latice occurs in several variants;

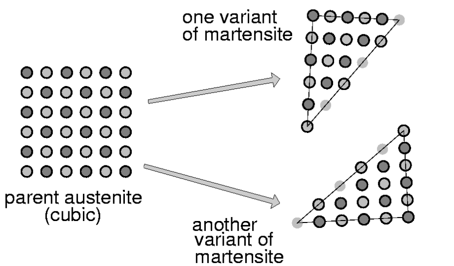


Shape-memory materials (SMM): alloys (=SMAs) or intermetalics.
The mechanism behind shape-memory effect (=SME):

◦ higher temperatures:
  atoms tend to form a latice with high symmetry (mostly cubic): austenite phase, higher heat capacity
  a lower-symmetrical latice: martensite phase, lower heat capacity.
  the lower-symmetrical latice occurs in several variants;

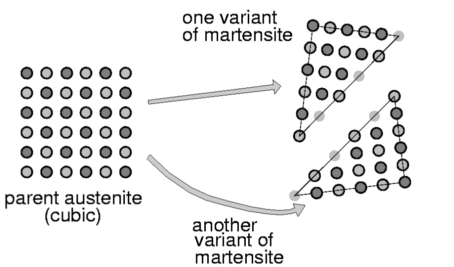


Shape-memory materials (SMM): alloys (=SMAs) or intermetalics.
The mechanism behind shape-memory effect (=SME):
∘ higher temperatures:
atoms tend to form a latice with high symmetry (mostly cubic):
austenite phase, higher heat capacity
a lower-symmetrical latice: martensite phase, lower heat capacity.
the lower-symmetrical latice occurs in several variants;
each of them can be rotated:

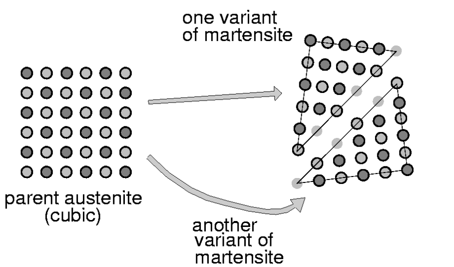


Shape-memory materials (SMM): alloys (=SMAs) or intermetalics.
The mechanism behind shape-memory effect (=SME):

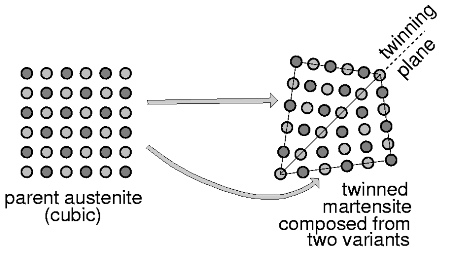
◦ higher temperatures:
  atoms tend to form a latice with high symmetry (mostly cubic):
  austenite phase, higher heat capacity
  a lower-symmetrical latice: martensite phase, lower heat capacity.
  the lower-symmetrical latice occurs in several variants;
  each of them can be rotated:



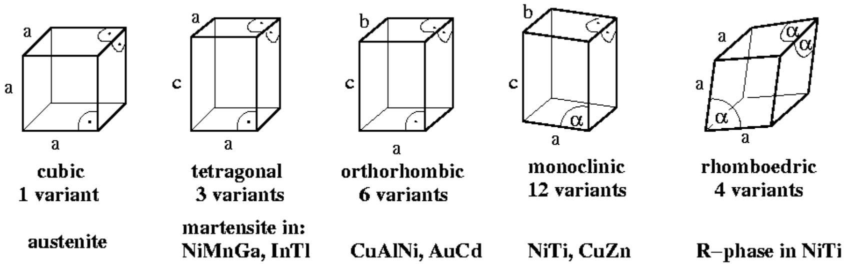
Shape-memory materials (SMM): alloys (=SMAs) or intermetalics.
The mechanism behind shape-memory effect (=SME):
◦ higher temperatures:
atoms tend to form a latice with high symmetry (mostly cubic): austenite phase, higher heat capacity
a lower-symmetrical latice: martensite phase, lower heat capacity.
the lower-symmetrical latice occurs in several variants;
each of them can be rotated:



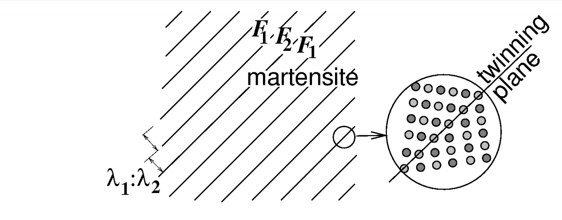
Shape-memory materials (SMM): alloys (=SMAs) or intermetalics.
The mechanism behind shape-memory effect (=SME):
◦ higher temperatures:
atoms tend to form a latice with high symmetry (mostly cubic):
austenite phase, higher heat capacity
a lower-symmetrical latice: martensite phase, lower heat capacity.
the lower-symmetrical latice occurs in several variants;
each of them can be rotated:



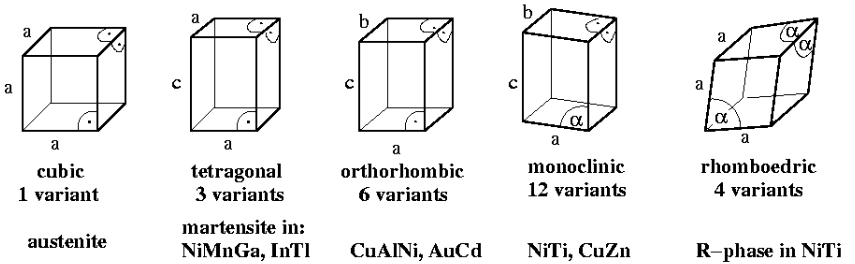
Crystalographical options of lower-symmetrical martensite:



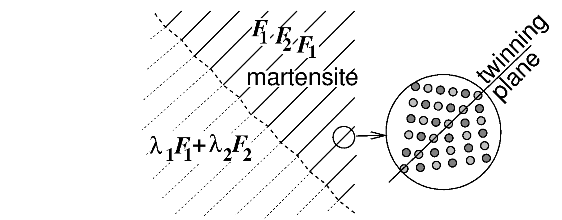
Self-accomodation of a microstructure in martensite



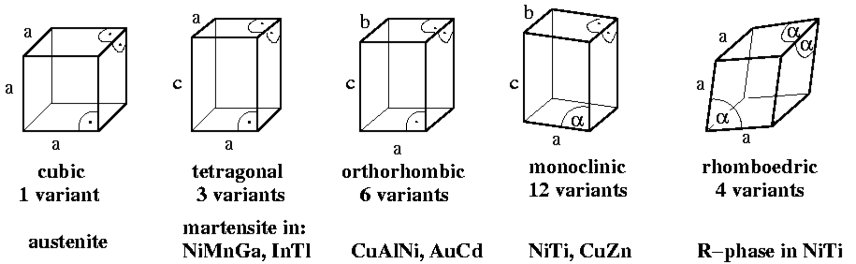
Crystalographical options of lower-symmetrical martensite:



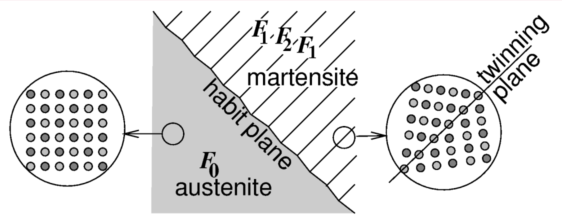
Self-accomodation of a microstructure in martensite



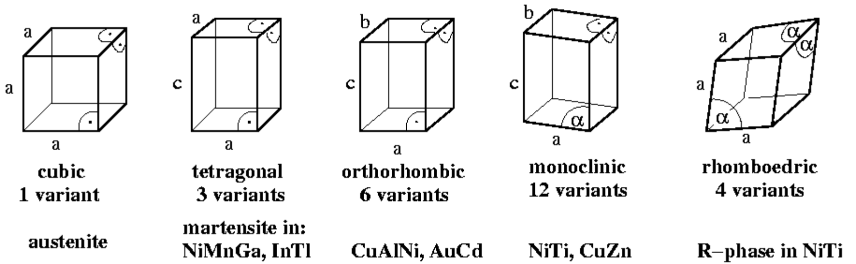
Crystalographical options of lower-symmetrical martensite:



Self-accomodation of a microstructure in austenite and martensite

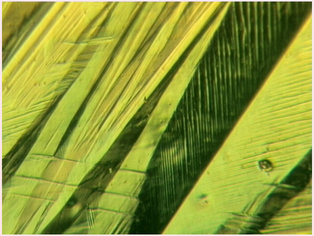


Crystalographical options of lower-symmetrical martensite:



Self-accomodation of a microstructure (example of CuAlNi)

.

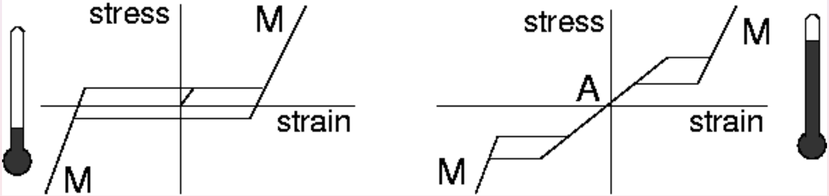


Courtesy of

Václav Novák and Petr Šittner,
Institute of Physics,
Academy of Sciences, Czech Rep.

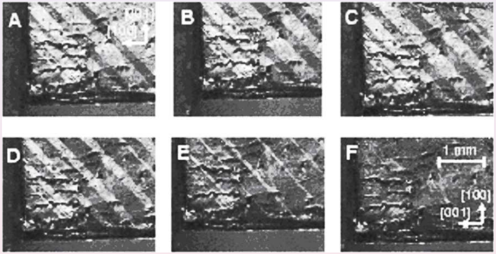
Schematic stress/strain response of SMM:

low temperature vs high temperature

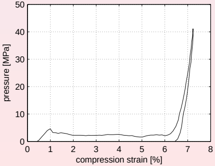


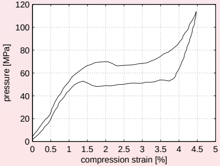
quasiplasticity pseudoelasticity

Experiments by L.Straka, V.Novák, M.Landa, O.Heczko, 2004:
Compression experiment: reorientation of tetragonal martensite in a (001)-oriented singlecrystal NiMnGa under temperature 293 K:



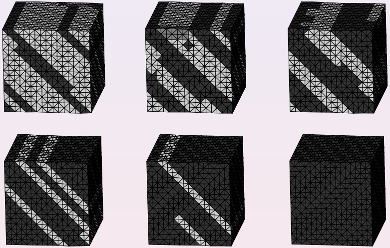
Stress-strain diagram at temperature 293 K (left) and 323 K (right):



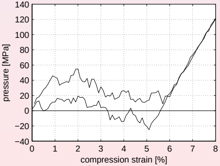


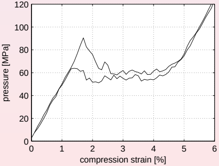
Computational simulations:

Compression experiment with NiMnGa (001)-oriented singlecrystal



Reorientation of martensite during a compression experiment at 293 K.



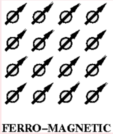


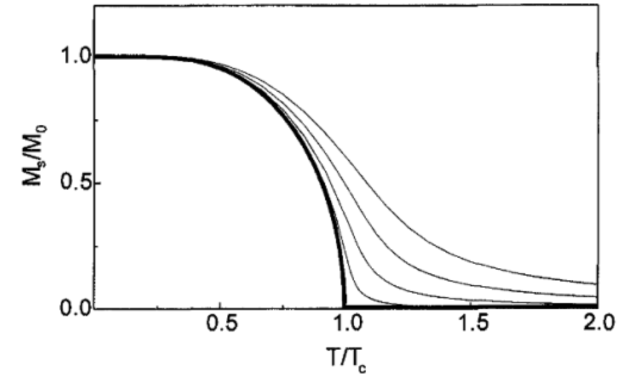
Stress/strain response during a compression experiment at 293 K and at 323 K.

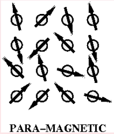
Calculations, visualizations: courtesy of Marcel Arndt, Universität Bonn.

Transformation in magnetic materials:

low temperature (below Currie point): highly-ordered, ferromagnetic state
very low temperature: the Heissenberg constraint $|m| = M_s$ is well satisfied
but in higher temperatures the deviation from it can be large in outer field
high temperature (above Currie point T_c): dis-ordered, paramagnetic state







G.Bertotti: *Hysteresis in Magnetism*.
Academic Press, San Diego, 1998.

Both martensite/austenite and ferro/para-magnetic transformations are coupled:

Strong dependence of thermo-mechanical response on magnetic field in Ni_2MnGa single crystals – for example:

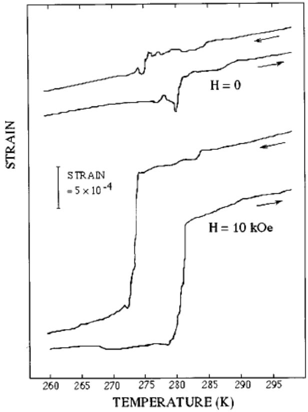


FIG. 2. Strain vs temperature in zero field and in 10 kOe. The two curves have been displaced relative to each other along the strain axis for clarity.

K.Ullakko, J.K.Huang, C.Kantner, R.C.O'Handley, V.V.Kokorin in *Appl. Phys. Lett.* **69** (1996), 1966–1968.

Other phenomena to be captured:

electric resistivity depending on temperature and phase (an example in NiTi):

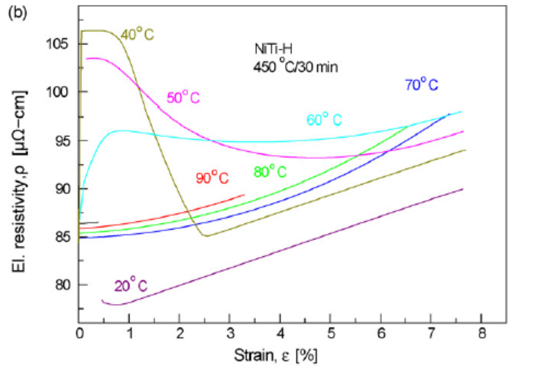


Fig. 5. Simulation: tensile stress–strain curves (a) and corresponding electrical resistivity changes (b) simulated for NiTi-H wire during tensile loading at various temperatures after cooling down from temperature $T = 80\,°C$ (austenite phase is stable) at zero stress.

V.Novák, P.Šittner, G.N.Dayananda, F.M.Braz-Fernandes, K.K.Mahesh, *Materials Science and Engineering A* **481-482** (2008) 127-133.

Variables (minimal scenario):

u displacement, $\quad \mathrm{E}(u) = \frac{1}{2}(\nabla u)^\top + \frac{1}{2}\nabla u =$ small-strain tensor,
m magnetisation,
θ temperature,
h magnetic field,
e electric field.

Basic concepts: small strains, Kelvin-Voigt rheology, 2nd-grade materials,
electric displacement current ($\sim$ electric-field energy) neglected,
$\Rightarrow$ eddy-current approximation of the Maxwell equations,
partly linear free energy $\varphi(\mathrm{E}, m, \theta) = \varphi_0(\mathrm{E}, m) + \theta\varphi_1(\mathrm{E}, m)$:
$\Rightarrow$ heat capacity $c = -\varphi''_{\theta\theta} = -\varphi''_{\theta\theta}(\theta)$,
cross-effects neglected (no Peltier/Seeback effects).

Main parameters of the model:

$\mathbb{K} = \mathbb{K}(\mathrm{E}, m, \theta)$ thermal conductivity,
$\mathbb{S} = \mathbb{S}(\mathrm{E}, m, \theta)$ electrical conductivity, $\quad c = c(\theta)$ heat capacity,
$\gamma = \gamma(|m|)$ effective gyromagnetic ratio, $\quad \mu_0$ vacuum permeability,
α magnetic-dissipation constant, $\quad \lambda$ magnetic exchange-energy constant,
ϱ mass density, $\quad f_0$ bulk force (inertial and load),
$\mathbb{D}$ viscosity tensor,
$\mathbb{C}_{\mathrm{H}}$ hyperelasticity tensor, $\quad \mathbb{D}_{\mathrm{H}}$ hyperviscosity tensor.

The equations:

Momentum equilibrium:

$$\varrho\ddot{u} - \mathrm{div}\Big(\varphi'_{\mathsf{E}}(\mathsf{E}(u),m,\theta) + \mathbb{D}\mathsf{E}(\dot{u}) - \mathrm{div}\big(\mathbb{C}_{\mathrm{H}}\nabla\mathsf{E}(u) + \mathbb{D}_{\mathrm{H}}\nabla\mathsf{E}(\dot{u})\big)\Big) = f_0 - \mu_0\nabla h^\top m,$$

Landau-Lifshitz-Gilbert equation:

$$\alpha\dot{m} - \frac{m\times\dot{m}}{\gamma(|m|)} - \lambda\,\Delta m + \varphi'_m(\mathsf{E}(u),m,\theta) = \mu_0 h,$$

heat equation

$$c(\theta)\dot{\theta} - \mathrm{div}\big(\mathbb{K}(\mathsf{E}(u),m,\theta)\nabla\theta\big) = \mathbb{S}(\mathsf{E}(u),m,\theta)e{:}e + \mathbb{D}\mathsf{E}(\dot{u}){:}\mathsf{E}(\dot{u}) + \mathbb{D}_{\mathrm{H}}\nabla\mathsf{E}(\dot{u})\vdots\nabla\mathsf{E}(\dot{u})$$
$$+\,\alpha|\dot{m}|^2 + \theta\varphi''_{\mathsf{E}\theta}(\mathsf{E}(u),m,\theta){:}\mathsf{E}(\dot{u}) + \theta\varphi''_{m\theta}(\mathsf{E}(u),m,\theta)\cdot\dot{m},$$

Maxwell system (in eddy-current approximation):

$$\mu_0(\dot{h}+\dot{m}) + \mathrm{curl}\ e = -\mu_0(\mathrm{div}\,\dot{u})m - \mu_0\nabla m\,\dot{u},$$
$$\varepsilon_0\dot{e} - \mathrm{curl}\ h + \mathbb{S}(\mathsf{E}(u),m,\theta)e = 0.$$

The equations:

Momentum equilibrium:

$$\varrho\ddot{u} - \mathrm{div}\Big(\varphi'_{\mathsf{E}}(\mathsf{E}(u),m,\theta) + \mathbb{D}\mathsf{E}(\dot{u}) - \mathrm{div}\big(\mathbb{C}_{\mathrm{H}}\nabla\mathsf{E}(u) + \mathbb{D}_{\mathrm{H}}\nabla\mathsf{E}(\dot{u})\big)\Big) = f_0 - \mu_0\nabla h^{\top} m,$$

Landau-Lifshitz-Gilbert equation:

$$\alpha\dot{m} - \frac{m\times\dot{m}}{\gamma(|m|)} - \lambda\Delta m + \varphi'_m(\mathsf{E}(u),m,\theta) = \mu_0 h,$$

heat equation

$$c(\theta)\dot{\theta} - \mathrm{div}\big(\mathbb{K}(\mathsf{E}(u),m,\theta)\nabla\theta\big) = \mathbb{S}(\mathsf{E}(u),m,\theta)e{:}e + \mathbb{D}\mathsf{E}(\dot{u}){:}\mathsf{E}(\dot{u}) + \mathbb{D}_{\mathrm{H}}\nabla\mathsf{E}(\dot{u}) \vdots \nabla\mathsf{E}(\dot{u}) + \alpha|\dot{m}|^2 + \theta\varphi''_{\mathsf{E}\theta}(\mathsf{E}(u),m,\theta){:}\mathsf{E}(\dot{u}) + \theta\varphi''_{m\theta}(\mathsf{E}(u),m,\theta)\cdot\dot{m},$$

Maxwell system (in eddy-current approximation):

$$\mu_0(\dot{h} + \dot{m}) + \mathrm{curl}\; e = -\mu_0(\mathrm{div}\,\dot{u})m - \mu_0\nabla m\,\dot{u},$$

$$\varepsilon_0\dot{e} - \mathrm{curl}\; h + \mathbb{S}(\mathsf{E}(u),m,\theta)e = 0.$$

The equations: ...analysed for slow loading $\Rightarrow$ pinning terms still needed

Momentum equilibrium: K.R.Rajagopal + T.R., 2003

$$\varrho\ddot{u} - \mathrm{div}\Big(\varphi'_{\mathrm{E}}(\mathrm{E}(u),m,\theta) + \mathbb{D}\mathrm{E}(\dot{u}) - \mathrm{div}\big(\mathbb{C}_{\mathrm{H}}\nabla\mathrm{E}(u) + \mathbb{D}_{\mathrm{H}}\nabla\mathrm{E}(\dot{u})\big)\Big) = f_0 - \mu_0\nabla h^\top m,$$

Landau-Lifshitz-Gilbert equation:

$$\alpha\dot{m} - \frac{m\times\dot{m}}{\gamma(|m|)} - \lambda\Delta m + \varphi'_m(\mathrm{E}(u),m,\theta) = \mu_0 h,$$

heat equation

$$c(\theta)\dot{\theta} - \mathrm{div}\big(\mathbb{K}(\mathrm{E}(u),m,\theta)\nabla\theta\big) = \mathbb{S}(\mathrm{E}(u),m,\theta)e{:}e + \mathbb{D}\mathrm{E}(\dot{u}){:}\mathrm{E}(\dot{u}) + \mathbb{D}_{\mathrm{H}}\nabla\mathrm{E}(\dot{u})\vdots\nabla\mathrm{E}(\dot{u})$$
$$+\alpha|\dot{m}|^2 + \theta\varphi''_{\mathrm{E}\theta}(\mathrm{E}(u),m,\theta){:}\mathrm{E}(\dot{u}) + \theta\varphi''_{m\theta}(\mathrm{E}(u),m,\theta)\cdot\dot{m},$$

Maxwell system (in eddy-current approximation):

$$\mu_0(\dot{h}+\dot{m}) + \mathrm{curl}\ e = -\mu_0(\mathrm{div}\dot{u})m - \mu_0\nabla m\,\dot{u},$$

$$\varepsilon_0\dot{e} - \mathrm{curl}\ h + \mathbb{S}(\mathrm{E}(u),m,\theta)e = 0.$$

Derivation of the Maxwell system:

m_s = magnetisation in the physical space,
"magnetic" part of the Maxwell system:

$$\mu_0(\dot{h} + \dot{m}_s) + \mathrm{curl}\, e = 0$$

m = magnetisation in the reference configuration related with m_s by

$$\det(\mathbb{I}+\nabla u(t,x))m_s(t,x+u(t,x)) = m(t,x).$$

Differentiation in time:

$$\det(\mathbb{I}+\nabla u)\big(\dot{m}_s + (\mathbb{I}+\nabla u)^{-\top}(\mathrm{div}\dot{u})m_s + \nabla m_s \dot{u}\big) = \dot{m}.$$

Small-displacement approximation $x + u \approx x$, which entails:
$\mathbb{I}+\nabla u \approx \mathbb{I}$, $m_s \approx m$, and $\nabla m_s \approx \nabla m$, so that

$$\dot{m}_s \approx \dot{m} - (\mathrm{div}\dot{u})m - \nabla m\, \dot{u} \qquad \longleftarrow \text{ occuring as r.h.s. of the Maxwell system}$$

$\dot{u}$ is not considered small
$\Rightarrow$ small but very fast mechanical vibrations
in some experiments on frequencies about 1 MHz or more.

Energetics:

Test: momentum eq. by $\dot{u}$, LLG by $\dot{m}$, heat eq. by 1, Maxwell by (h, e):

Use: cancelation of the gyromagnetic term: $\frac{m\times\dot{m}}{\gamma(|m|)}\cdot\dot{m}=0$,

$+$ cancelation of curl-terms $+$ the identity:

$$\int_\Omega \underbrace{\mu_0\big((\mathrm{div}\dot{u})m+\nabla m\,\dot{u}\big)}_{\text{r.h.s. of Maxwell eq.}}\cdot h\,\mathrm{d}x=\int_\Omega \mu_0\mathrm{div}(m\otimes\dot{u})\cdot h\,\mathrm{d}x$$
$$=\int_\Omega \mu_0\left(\mathrm{div}((\dot{u}\otimes m)h)-(m\otimes\dot{u}):\nabla h\right)\mathrm{d}x$$
$$=\int_\Gamma \mu_0(m\cdot h)(\dot{u}\cdot n)\,\mathrm{d}S-\int_\Omega \underbrace{\mu_0\nabla h^\top m}_{\substack{\text{r.h.s. of}\\ \text{momentum equation}}}\cdot\dot{u}\,\mathrm{d}x$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_\Omega \underbrace{\varepsilon}_{\substack{\text{internal}\\ \text{energy}}}+\underbrace{\frac{\mu_0}{2}|h|^2}_{\substack{\text{magnetic}\\ \text{energy}}}+\underbrace{\frac{\varrho}{2}|\dot{u}|^2}_{\substack{\text{kinetic}\\ \text{energy}}}\,\mathrm{d}x=\int_\Omega \underbrace{f_0\cdot\dot{u}}_{\substack{\text{power of}\\ \text{external load}}}\,\mathrm{d}x+\text{ boundary power.}$$

Gibbs' relation: $\psi=\varepsilon-s\theta$ with entropy $s=-\phi'_\theta$

internal energy: $\varepsilon=\vartheta+\psi(\mathrm{E},m,0)+\frac{1}{2}\mathbb{C}_{\mathrm{H}}\nabla\mathrm{E}\vdots\nabla\mathrm{E}+\frac{1}{2}\lambda|\nabla m|^2$, enthapy $\vartheta=\int_0^\theta c(\cdot)$.

Example of a free energy considered in NiMnGa:

described by the magnetization M. A free-energy expansion is formulated in such a way that the functional is invariant under the actions $\hat{O}$ of the space group of the fcc phase of Ni-Mn-Ga ($\hat{O}(F) \to F$, $\hat{O} \in O_h$), leading to

$$F = \frac{1}{2}(c_{11} + 2c_{12})e_1^2 + \frac{1}{2}a(e_2^2 + e_3^2) + \frac{1}{2}c_{44}(e_4^2 + e_5^2 + e_6^2) + \frac{1}{3}be_3(e_3^2 - 3e_2^2)$$
$$+\frac{1}{4}c(e_2^2 + e_3^2)^2 + \frac{1}{\sqrt{3}}B_1e_1\mathbf{m}^2 + B_2\left[\frac{1}{\sqrt{2}}e_2(m_x^2 - m_y^2) + \frac{1}{\sqrt{6}}e_3(3m_z^2 - \mathbf{m}^2)\right]$$
$$+B_3\left(e_4m_xm_y + e_5m_ym_z + e_6m_zm_x\right) + K_1(m_x^2m_y^2 + m_y^2m_z^2 + m_x^2m_z^2)$$
$$+\frac{1}{2}\alpha\mathbf{m}^2 + \frac{1}{4}\delta_1\mathbf{m}^4 - \mathbf{M_0H_0}, \tag{1}$$

where the e_i are linear combinations of the strain tensor components e_{ik},

$$\begin{aligned} e_1 &= (e_{xx} + e_{yy} + e_{zz})/\sqrt{3}, \\ e_2 &= (e_{xx} - e_{yy})/\sqrt{2}, \\ e_3 &= (2e_{zz} - e_{xx} - e_{yy})/\sqrt{6}, \\ e_4 &= e_{xy},\ e_5 = e_{yz},\ e_6 = e_{zx}. \end{aligned} \tag{2}$$

In (1), a, b and c are linear combinations of the components of the second, third and fourth order elasticity moduli, respectively, with $a = c_{11} - c_{12}$, $b = (c_{111} - 3c_{112} + 2c_{123})/6\sqrt{6}$ and $c = (c_{1111} + 6c_{1112} - 3c_{1122} - 8c_{1123})/48$ (Fradkin, 1994); $\mathbf{m} = \mathbf{M}/M_0$ is the unit vector of the magnetization and M_0 saturation magnetization; B_i are magnetostriction constants; K_1 is the first cubic anisotropy constant; α_1 and δ_1 are exchange parameters.

in partly linearized ansatz:

critical points T_C and T_M the parameters a and α can be expressed as

$$a = a_0(T - T_M), \quad \alpha = \alpha_0(T - T_C),$$

A.T.Zayak, V.D.Buchelnikov, P.Entel: A Ginzburg-Landau theory for Ni-Mn-Ga. *Phase Trans.* **75** (2002), 243-256

Fully implicit time-discretisation + regularization:

Recursive formula for the 5-tuple $(u_\tau^k, m_\tau^k, \vartheta_\tau^k, e_\tau^k, h_\tau^k)$ solving the system

Momentum-equilibrium equation:

$$\varrho\frac{u_\tau^k-2u_\tau^{k-1}+u_\tau^{k-2}}{\tau^2}-\mathrm{div}\Big(S_\tau^k-\mathrm{div}\,H_\tau^k\Big)=f_\tau^k-\mu_0(\nabla h_\tau^k)^\top m_\tau^k \quad \text{with}$$

$$S_\tau^k:=\sigma_{\mathsf E}(\mathsf E(u_\tau^k),m_\tau^k,\vartheta_\tau^k)+\mathbb D\mathsf E\Big(\frac{u_\tau^k-u_\tau^{k-1}}{\tau}\Big)+\tau|\mathsf E(u_\tau^k)|^{\eta-2}\mathsf E(u_\tau^k), \text{ and}$$

$$H_\tau^k:=\mathbb D_{\rm H}\nabla\mathsf E\Big(\frac{u_\tau^k-u_\tau^{k-1}}{\tau}\Big)+\mathbb C_{\rm H}\nabla\mathsf E(u_\tau^k)+\tau|\nabla\mathsf E(u_\tau^k)|^{\eta-2}\nabla\mathsf E(u_\tau^k),$$

Landau-Lifshitz-Gilbert equation:

$$\alpha\frac{m_\tau^k-m_\tau^{k-1}}{\tau}-\frac{m_\tau^k}{\gamma(|m_\tau^k|)}\times\frac{m_\tau^k-m_\tau^{k-1}}{\tau}-\lambda\Delta m_\tau^k+\sigma_m(\mathsf E(u_\tau^k),m_\tau^k,\vartheta_\tau^k)$$
$$-\mu_0h_\tau^k=\tau\mathrm{div}\big(|\nabla m_\tau^k|^{\eta-2}\big)\nabla m_\tau^k\big)-\tau|m_\tau^k|^{\eta-2}m_\tau^k,$$

Heat equation:

$$\frac{\vartheta_\tau^k - \vartheta_\tau^{k-1}}{\tau} - \mathrm{div}\Big(K_0(\mathsf{E}_\tau^k, m_\tau^k, \vartheta_\tau^k)\nabla\vartheta_\tau^k\Big) = S(\mathsf{E}_\tau^k, m_\tau^k, \vartheta_\tau^k)e_\tau^k{:}e_\tau^k$$
$$+\Big(1-\frac{\sqrt{\tau}}{2}\Big)\mathbb{D}\mathsf{E}\Big(\frac{u_\tau^k-u_\tau^{k-1}}{\tau}\Big){:}\mathsf{E}\Big(\frac{u_\tau^k-u_\tau^{k-1}}{\tau}\Big)$$
$$+\mathbb{D}_{\mathrm{H}}\nabla\mathsf{E}\Big(\frac{u_\tau^k-u_\tau^{k-1}}{\tau}\Big)\vdots\nabla\mathsf{E}\Big(\frac{u_\tau^k-u_\tau^{k-1}}{\tau}\Big)$$
$$+\Big(1-\frac{\sqrt{\tau}}{2}\Big)\alpha\Big|\frac{m_\tau^k-m_\tau^{k-1}}{\tau}\Big|^2 + A\Big(\mathsf{E}_\tau^k, m_\tau^k, \vartheta_\tau^k; \frac{\mathsf{E}_\tau^k-\mathsf{E}_\tau^{k-1}}{\tau}, \frac{m_\tau^k-m_\tau^{k-1}}{\tau}\Big),$$

Maxwell system:

$$\frac{h_\tau^k-h_\tau^{k-1}}{\tau} + \frac{\mathrm{curl}\, e_\tau^k}{\mu_0} = \nabla m_\tau^k \frac{u_\tau^k-u_\tau^{k-1}}{\tau} - \frac{m_\tau^k-m_\tau^{k-1}}{\tau} + \mathrm{div}\frac{u_\tau^k-u_\tau^{k-1}}{\tau} m_\tau^k,$$
$$\mathrm{curl}\, h_\tau^k - S(\mathsf{E}_\tau^k, m_\tau^k, \vartheta_\tau^k)\, e_\tau^k = \tau|e_\tau^k|^{\eta-2}e_\tau^k,$$

---

for $k = 1, ..., K_\tau := T/\tau$, where we abbreviated $\mathsf{E}_\tau^k = \mathsf{E}(u_\tau^k)$ and
$A(\mathsf{E}, m, \vartheta; \dot{\mathsf{E}}, \dot{m}) = \theta\varphi''_{\mathsf{E}\theta}(\mathsf{E}, m, \theta){:}\dot{\mathsf{E}} + \theta\varphi''_{m\theta}(\mathsf{E}, m, \theta)\cdot\dot{m}$
with $\theta = \widehat{c}^{-1}(\vartheta)$ and $\widehat{c}' = c$.

We use the discrete scheme recursively, starting from $k=1$ by using

$$u_\tau^0=u_{0,\tau},\qquad u_\tau^{-1}=u_{0,\tau}-\tau v_0,\qquad m_\tau^0=m_{0,\tau},\qquad \vartheta_\tau^0=\hat{c}(\theta_0),\qquad h_\tau^0=h_0,$$

Existence of $(u_\tau^k,m_\tau^k,\vartheta_\tau^k,e_\tau^k,h_\tau^k)$:

η large enough (namely $\eta>8$) $\Rightarrow$ pseudomonotone coercive operator
$\Rightarrow$ Brézis' theorem $\Rightarrow$

$$\begin{aligned}
&u_\tau^k\in W^{2,\eta}(\Omega;\mathbb{R}^3),\\
&m_\tau^k\in W^{1,\eta}(\Omega;\mathbb{R}^3),\\
&\vartheta_\tau^k\in W^{1,2}(\Omega),\\
&h_\tau^k\in L_{\rm curl}^{2,\eta'}(\Omega;\mathbb{R}^3),\\
&e_\tau^k\in L_{\rm curl}^{\eta,2}(\Omega;\mathbb{R}^3)
\end{aligned}$$

where $L_{\rm curl}^{p,q}(\Omega;\mathbb{R}^3):=\big\{v\in L^p(\Omega;\mathbb{R}^3);\ {\rm curl}\ v\in L^q(\Omega;\mathbb{R}^3)\big\}$.

Non-negativity: $\vartheta_\tau^k\geq 0$.

A-priori estimates (u_τ, $\bar{u}_\tau$ etc. are interpolants over $[0,T]$):

Energy-type test (by $\dot{u}_\tau$, $\dot{m}_\tau$, 1, $\bar{h}_\tau$, $\bar{e}_\tau$) $\Rightarrow$

$$\left\|u_\tau\right\|_{W^{1,\infty}(I;L^2(\Omega;\mathbb{R}^3))\cap W^{1,2}(I;W^{2,2}(\Omega;\mathbb{R}^3))} \le C,$$

$$\left\|m_\tau\right\|_{L^\infty(I;W^{1,2}(\Omega;\mathbb{R}^3))\cap W^{1,2}(I;L^2(\Omega;\mathbb{R}^3))} \le C,$$

$$\left\|\bar{\vartheta}_\tau\right\|_{L^\infty(I;L^1(\Omega))} \le C,$$

$$\left\|h_\tau\right\|_{L^\infty(I;L^2(\Omega;\mathbb{R}^3))} \le C,$$

$$\left\|\bar{e}_\tau\right\|_{L^2(Q;\mathbb{R}^3)} \le C,$$

$$\left\|\mathrm{E}(\bar{u}_\tau)\right\|_{L^\infty(I;W^{1,\eta}(\Omega;))} \le C\tau^{-1/\eta},$$

$$\left\|m_\tau\right\|_{L^\infty(I;W^{1,\eta}(\Omega;\mathbb{R}^3))} \le C\tau^{-1/\eta},$$

$$\left\|e_\tau\right\|_{L^\eta(Q;\mathbb{R}^3)} \le C\tau^{-1/\eta}$$

based on the semi-convexity of the functional (for enough small $\tau > 0$)

$$(\mathrm{E},m) \mapsto \varphi(\mathrm{E},m) + \frac{\tau}{\eta}|\mathrm{E}|^\eta + \frac{\tau}{\eta}|m|^\eta + \frac{\mathbb{D}\mathrm{E}{:}\mathrm{E} + \alpha|m|^2}{2\sqrt{\tau}}$$

A-priori estimates (u_τ, $\bar u_\tau$ etc. are interpolants over $[0,T]$):

Energy-type test (by $\dot u_\tau$, $\dot m_\tau$, 1, $\bar h_\tau$, $\bar e_\tau$) $\Rightarrow$

$$\big\|u_\tau\big\|_{W^{1,\infty}(I;L^2(\Omega;\mathbb{R}^3))\cap W^{1,2}(I;W^{2,2}(\Omega;\mathbb{R}^3))} \le C,$$
$$\big\|m_\tau\big\|_{L^\infty(I;W^{1,2}(\Omega;\mathbb{R}^3))\cap W^{1,2}(I;L^2(\Omega;\mathbb{R}^3))} \le C,$$
$$\big\|\bar\vartheta_\tau\big\|_{L^\infty(I;L^1(\Omega))} \le C,$$
$$\big\|h_\tau\big\|_{L^\infty(I;L^2(\Omega;\mathbb{R}^3))} \le C,$$
$$\big\|\bar e_\tau\big\|_{L^2(Q;\mathbb{R}^3)} \le C,$$
$$\big\|\mathrm{E}(\bar u_\tau)\big\|_{L^\infty(I;W^{1,\eta}(\Omega;))} \le C\tau^{-1/\eta},$$
$$\big\|m_\tau\big\|_{L^\infty(I;W^{1,\eta}(\Omega;\mathbb{R}^3))} \le C\tau^{-1/\eta},$$
$$\big\|e_\tau\big\|_{L^\eta(Q;\mathbb{R}^3)} \le C\tau^{-1/\eta}$$

based on the semi-convexity of the functional (for enough small $\tau>0$)

$$(\mathrm{E},m)\mapsto\varphi(\mathrm{E},m)+\frac{\tau}{\eta}|\mathrm{E}|^\eta+\frac{\tau}{\eta}|m|^\eta+\frac{\mathbb{D}\mathrm{E}{:}\mathrm{E}+\alpha|m|^2}{2\sqrt{\tau}}$$

A-priori estimates (u_τ, $\bar{u}_\tau$ etc. are interpolants over $[0,T]$):

Energy-type test (by $\dot{u}_\tau$, $\dot{m}_\tau$, 1, $\bar{h}_\tau$, $\bar{e}_\tau$) $\Rightarrow$

$$\begin{aligned}
&\big\|u_\tau\big\|_{W^{1,\infty}(I;L^2(\Omega;\mathbb{R}^3))\cap W^{1,2}(I;W^{2,2}(\Omega;\mathbb{R}^3))} \le C,\\
&\big\|m_\tau\big\|_{L^\infty(I;W^{1,2}(\Omega;\mathbb{R}^3))\cap W^{1,2}(I;L^2(\Omega;\mathbb{R}^3))} \le C,\\
&\big\|\bar{\vartheta}_\tau\big\|_{L^\infty(I;L^1(\Omega))} \le C,\\
&\big\|h_\tau\big\|_{L^\infty(I;L^2(\Omega;\mathbb{R}^3))} \le C,\\
&\big\|\bar{e}_\tau\big\|_{L^2(Q;\mathbb{R}^3)} \le C,\\
&\big\|\mathrm{E}(\bar{u}_\tau)\big\|_{L^\infty(I;W^{1,\eta}(\Omega;))} \le C\tau^{-1/\eta},\\
&\big\|m_\tau\big\|_{L^\infty(I;W^{1,\eta}(\Omega;\mathbb{R}^3))} \le C\tau^{-1/\eta},\\
&\big\|e_\tau\big\|_{L^\eta(Q;\mathbb{R}^3)} \le C\tau^{-1/\eta},
\end{aligned}$$

$+$ a special nonlinear test of the heat equation $+$ Gagliardo-Nirenberg interpolation

$$\big\|\nabla\bar{\vartheta}_\tau\big\|_{L^r(Q;\mathbb{R}^d)} \le C_r \qquad \text{with} \quad r < 5/4.$$

Further a-priori estimates:

$$\Big\|\varrho\,\ddot{u}^{\rm i}_\tau\Big\|_{L^2(I;W^{2,2}(\Omega;\mathbb{R}^3)^*)+L^{\eta'}(I;W^{2,\eta}(\Omega;\mathbb{R}^3)^*)}\le C,$$

$$\Big\|\varrho\,\ddot{u}^{\rm i}_\tau-\tau{\rm div}\big(|\mathsf{E}(\bar u_\tau)|^{\eta-2}\mathsf{E}(\bar u_\tau)\big)+\tau{\rm div}^2\big(|\nabla\mathsf{E}(\bar u_\tau)|^{\eta-2}\nabla\mathsf{E}(\bar u_\tau)\big)\Big\|_{L^2(I;W^{2,2}(\Omega;\mathbb{R}^3)^*)}\le C,$$

$$\big\|\dot\vartheta_\tau\big\|_{L^1(I;W^{3,2}(\Omega)^*)}\le C,$$

$$\big\|{\rm curl}\,\bar h_\tau+\tau|\bar e_\tau|^{\eta-2}\bar e_\tau\big\|_{L^2(Q;\mathbb{R}^3)}\le C,$$

$$\big\|\dot h_\tau\big\|_{L^2(I;L^2_{{\rm curl},0}(\Omega;\mathbb{R}^3)^*)}\le C.$$

Convergence for $\tau \to 0$: Step 0: Banach' selection principle:

$$
\begin{aligned}
&u_\tau \to u && \text{strongly in } W^{1,2}(I;W^{2,2}(\Omega;\mathbb{R}^3)),\\
&m_\tau \to m && \text{strongly in } W^{1,2}(I;W^{1,2}(\Omega;\mathbb{R}^3)),\\
&\bar\vartheta_\tau \to \vartheta && \text{strongly in } L^s(Q) \text{ with any } s<5/3,\\
&\bar e_\tau \to e && \text{strongly in } L^2(Q;\mathbb{R}^3),\\
&\bar h_\tau \to h && \text{weakly* in } L^\infty(I;L^2(\Omega;\mathbb{R}^3)),
\end{aligned}
$$

and, moreover (with $h_{\rm b}$ from not-mentioned boundary conditions)

$$
\begin{aligned}
&\bar h_\tau - \bar h_{{\rm b},\tau} \to h - h_{\rm b} && \text{weakly in } L^{\eta'}(I;L^{2,\eta'}_{{\rm curl},0}(\Omega;\mathbb{R}^3)), \text{ and}\\
&{\rm curl}\,\bar h_\tau + \tau|\bar e_\tau|^{\gamma-2}\bar e_\tau \to {\rm curl}\,h && \text{weakly in } L^2(Q;\mathbb{R}^{3\times 3}).
\end{aligned}
$$

for a subsequence.

Then we want to prove that any (u, m, ϑ, h, e) obtained in this way is a weak solution to the considered IBVP (after the transformation $\theta \mapsto \vartheta$) which also preserves the total energy.

Step 1: Convergence in the semilinear mechanical/magnetic/electro part:
Aubin-Lions' theorem: strong convergence of $\mathrm{E}(\bar{u}_\tau)$, $\bar{m}_\tau$, and $\bar{\vartheta}_\tau$.
Then weak convergence suffices in semilinear terms, while
the quasilinear regularizing terms vanish, e.g.

$$\Big|\int_Q \tau|\mathrm{E}(\bar{u}_\tau)|^{\eta-2}\mathrm{E}(\bar{u}_\tau){:}\mathrm{E}(v)\,\mathrm{d}x\mathrm{d}t\Big| \le \tau\|\mathrm{E}(\bar{u}_\tau)\|^{\eta-1}_{L^\eta(Q;\mathbb{R}^{3\times3})}\|\mathrm{E}(v)\|_{L^\eta(Q;\mathbb{R}^{3\times3})} \le C\tau^{1/\eta}\|\mathrm{E}(v)\|_{L^\eta(Q;\mathbb{R}^{3\times3})} \to 0$$

for any smooth v.

Step 2: Mechanical/magnetic energy preservation:
test respectively by $\dot{u}$, $\dot{m}$, h, and e and make the by-part integration

$$\varrho\,\ddot{u}^{\mathrm{i}}_\tau - \tau\mathrm{div}\big(|\mathrm{E}(\bar{u}_\tau)|^{\eta-2}\mathrm{E}(\bar{u}_\tau)\big) + \tau\mathrm{div}^2\big(|\nabla\mathrm{E}(\bar{u}_\tau)|^{\eta-2}\nabla\mathrm{E}(\bar{u}_\tau)\big) \rightharpoonup \zeta \in L^2(I;W^{2,2}(\Omega;\mathbb{R}^3)^*)$$

and then

$$\langle\zeta, w\rangle = \lim_{\tau\to0}\int_Q \varrho\dot{u}^{\mathrm{i}}_\tau\cdot\dot{w} - \tau|\mathrm{E}(\bar{u}_\tau)|^{\eta-2}\mathrm{E}(\bar{u}_\tau):\mathrm{E}(w) + \tau|\nabla\mathrm{E}(\bar{u}_\tau)|^{\eta-2}\nabla\mathrm{E}(\bar{u}_\tau)\vdots\nabla\mathrm{E}(w)\mathrm{d}x\mathrm{d}t = \int_Q \varrho\dot{u}\cdot\dot{w}\mathrm{d}x\mathrm{d}t.$$

$\Rightarrow\quad \zeta = \varrho\ddot{u} \quad\Rightarrow\quad \varrho\ddot{u}$ is in duality with $\dot{u} \in L^2(I;W^{2,2}(\Omega;\mathbb{R}^3))$.

Step 3: Strong convergence of $\nabla \mathrm{E}(\dot{u}_\tau)$ and $\dot{m}_\tau$ and $\bar{e}_\tau$:

$$\int_Q \mathbb{D}\mathrm{E}(\dot{u}){:}\mathrm{E}(\dot{u}) + \mathbb{D}_{\mathrm{H}}\nabla\mathrm{E}(\dot{u}){:}\nabla\mathrm{E}(\dot{u}) + \alpha|\dot{m}|^2 + S(\mathrm{E},m,\vartheta)e{\cdot}e$$

$$\leq \liminf_{\tau\to 0} \int_Q \mathbb{D}\mathrm{E}(\dot{u}_\tau){:}\mathrm{E}(\dot{u}_\tau) + \mathbb{D}_{\mathrm{H}}\nabla\mathrm{E}(\dot{u}_\tau){:}\nabla\mathrm{E}(\dot{u}_\tau) + \alpha|\dot{m}_\tau|^2 + S(\bar{\mathrm{E}}_\tau,\bar{m}_\tau,\bar{\vartheta}_\tau)\bar{e}_\tau{\cdot}\bar{e}_\tau \mathrm{d}x$$

$$\leq \limsup_{\tau\to 0} \int_Q \mathbb{D}\mathrm{E}(\dot{u}_\tau){:}\mathrm{E}(\dot{u}_\tau) + \mathbb{D}_{\mathrm{H}}\nabla\mathrm{E}(\dot{u}_\tau){:}\nabla\mathrm{E}(\dot{u}_\tau) + \alpha|\dot{m}_\tau|^2 + S(\bar{\mathrm{E}}_\tau,\bar{m}_\tau,\bar{\vartheta}_\tau)\bar{e}_\tau{\cdot}\bar{e}_\tau \mathrm{d}x$$

$$\leq \limsup_{\tau\to 0} \Phi\big(u_{0\tau},v_0,m_{0\tau},h_0\big) - \Phi\big(u_\tau(T),\dot{u}_\tau(T),m_\tau(T),h_\tau(T)\big)$$

$$+ \int_\Omega \frac{\tau}{\eta}|\mathrm{E}(u_{0\tau})|^\eta + \frac{\tau}{\eta}|\nabla\mathrm{E}(u_{0\tau})|^\eta + \frac{\tau}{\eta}|m_{0\tau}|^\eta \mathrm{d}x - \int_\Sigma \bar{g}_\tau{\cdot}\dot{u}_\tau \mathrm{d}t + \int_Q \bar{f}_\tau{\cdot}\dot{u}_\tau + \mathrm{curl}\, h_{\mathrm{b},\tau}{\cdot}e_\tau$$

$$+ \mu_0\big(\dot{h}_\tau + \dot{m}_\tau - \nabla\bar{m}_\tau\dot{u}_\tau - (\mathrm{div}\,\dot{u}_\tau)_\tau\big){\cdot}h_{\mathrm{b},\tau} - A(\bar{\mathrm{E}}_\tau,\bar{m}_\tau,\bar{\vartheta}_\tau;\dot{\mathrm{E}}_\tau,\dot{m}_\tau)\mathrm{d}x\mathrm{d}t$$

$$\leq \Phi\big(u_0,v_0,m_0,h_0\big) - \Phi\big(u(T),\dot{u}(T),m(T),h(T)\big) - \int_\Sigma g{\cdot}\dot{u}\mathrm{d}t + \int_Q f{\cdot}\dot{u} + \mathrm{curl}\, h_{\mathrm{b}}{\cdot}e$$

$$+ \mu_0\big(\dot{h} + \dot{m} - \nabla m\dot{u} - (\mathrm{div}\,\dot{u})m\big){\cdot}h_{\mathrm{b}} - A(\mathrm{E},m,\vartheta;\dot{\mathrm{E}},\dot{m})\mathrm{d}x\mathrm{d}t$$

$$= \int_Q \mathbb{D}\mathrm{E}(\dot{u}){:}\mathrm{E}(\dot{u}) + \mathbb{D}_{\mathrm{H}}\nabla\mathrm{E}(\dot{u}){:}\nabla\mathrm{E}(\dot{u}) + \alpha|\dot{m}|^2 + S(\mathrm{E},m,\vartheta)e{\cdot}e.$$

Step 4: Limit passage in the heat equation:
Having proved the strong convergence in Step 2, the right-hand side of the heat equation converges strongly in $L^1(Q)$ and this limit passage is then easy.

Step 5: Total energy preservation:
We have $\dot{\vartheta} \in L^1(I; W^{3,2}(\Omega)^*)$, and realize the already proved the heat equation, which is in duality with the constant 1, we can perform rigorously this test and sum it with mechanical/magnetic energy balance obtained already in Step 2.

Fully nonlinear coupling:

more symmetry $\sim$ higher heat capacity
heat capacity is higher in austenite than in martensite

$\Rightarrow$ shape-memory effect

c should depend rather directly on E (and also m, not only on θ)

fully general nonlinear ansatz $\varphi(\mathrm{E}, m, \theta)$ instead of $\varphi_0(\mathrm{E}, m) + \theta\varphi_1(\mathrm{E}, m)$

then the heat capacity $c = -\psi''_{\theta\theta}$ depends, beside θ, also on E and m.

Generalized enthalpy transformation:

$$\vartheta = \hat{c}(\mathrm{E}, m, \theta) := \int_0^\theta c(\mathrm{E}, m, \Theta)\mathrm{d}\Theta$$

$$c(\mathrm{E}, m, \theta)\dot{\theta} = \frac{\partial \hat{c}(\mathrm{E}, m, \theta)}{\partial t} - c_1(\mathrm{E}, m, \theta):\dot{\mathrm{E}} - c_2(\mathrm{E}, m, \theta)\cdot\dot{m},$$

$$c_1(\mathrm{E}, m, \theta) = \int_0^\theta c'_{\mathrm{E}}(\mathrm{E}, m, \Theta)\mathrm{d}\Theta \quad \text{and} \quad c_2(\mathrm{E}, m, \theta) = \int_0^\theta c'_m(\mathrm{E}, m, \Theta)\mathrm{d}\Theta.$$

Define: $\mathcal{T}(\mathrm{E},m,\cdot) := [\hat{c}(\mathrm{E},m,\cdot)]^{-1}$

$$
\begin{aligned}
&\mathcal{K}_0(\mathrm{E},m,\vartheta) := \mathbb{K}(\mathrm{E},m,\mathcal{T}(\mathrm{E},m,\vartheta))\mathcal{T}'_\vartheta(\mathrm{E},m,\vartheta),\\
&\mathcal{K}_1(\mathrm{E},m,\vartheta) := \mathbb{K}(\mathrm{E},m,\mathcal{T}(\mathrm{E},m,\vartheta))\mathcal{T}'_\mathrm{E}(\mathrm{E},m,\vartheta),\\
&\mathcal{K}_2(\mathrm{E},m,\vartheta) := \mathbb{K}(\mathrm{E},m,\mathcal{T}(\mathrm{E},m,\vartheta))\mathcal{T}'_m(\mathrm{E},m,\vartheta),\\
&\mathcal{S}(\mathrm{E},m,\vartheta) := \ \mathbb{S}(\mathrm{E},m,\mathcal{T}(\mathrm{E},m,\vartheta)),\\
&\mathcal{A}_1(\mathrm{E},m,\vartheta) := \mathcal{T}(\mathrm{E},m,\vartheta)\varphi''_{\mathrm{E}\theta}(\mathrm{E},m,\mathcal{T}(\mathrm{E},m,\vartheta)) + c_1(\mathrm{E},m,\mathcal{T}(\mathrm{E},m,\vartheta))\\
&\mathcal{A}_2(\mathrm{E},m,\vartheta) := \mathcal{T}(\mathrm{E},m,\vartheta)\varphi''_{m\theta}(\mathrm{E},m,\mathcal{T}(\mathrm{E},m,\vartheta)) + c_2(\mathrm{E},m,\mathcal{T}(\mathrm{E},m,\vartheta)),\\
&\sigma_\mathrm{E}(\mathrm{E},m,\vartheta) := \varphi'_\mathrm{E}(\mathrm{E},m,\mathcal{T}(\mathrm{E},m,\vartheta)),\\
&\sigma_m(\mathrm{E},m,\vartheta) := \varphi'_m(\mathrm{E},m,\mathcal{T}(\mathrm{E},m,\vartheta)).
\end{aligned}
$$

Then the heat flux transforms to:

$$
\begin{aligned}
\mathbb{K}(\mathrm{E},m,\theta)\nabla\theta &= \mathbb{K}(\mathrm{E},m,\mathcal{T}(e,\vartheta))\nabla\mathcal{T}(\mathrm{E},m,\vartheta)\\
&= \mathcal{K}_0(\mathrm{E},m,\vartheta)\nabla\vartheta + \mathcal{K}_1(\mathrm{E},m,\vartheta)\nabla\mathrm{E} + \mathcal{K}_2(\mathrm{E},m,\vartheta)\nabla m.
\end{aligned}
$$

Thus, in terms of the 5-tuple (u, m, ϑ, e, h), the original system transforms to the following 5 equations:

$$\varrho\,\ddot{u} - \mathrm{div}\Big(\sigma_{\mathsf{E}}(\mathsf{E}(u), m, \vartheta) + \mathbb{D}\mathsf{E}(\dot{u}) - \mathrm{div}\big(\mathbb{C}_{\mathrm{H}}\nabla\mathsf{E}(u) + \mathbb{D}_{\mathrm{H}}\nabla\mathsf{E}(\dot{u})\big)\Big) = f_0 - \mu_0\nabla h^\top m,$$

$$\alpha\dot{m} - \frac{m\times\dot{m}}{\gamma(|m|)} - \lambda\,\Delta m + \sigma_m(\mathsf{E}(u), m, \vartheta) = p_0 + \mu_0 h,$$

$$\dot{\vartheta} - \mathrm{div}\Big(\mathcal{K}_0(\mathsf{E}(u), m, \vartheta)\nabla\vartheta + \mathcal{K}_1(\mathsf{E}(u), m, \vartheta)\nabla\mathsf{E}(u) + \mathcal{K}_2(\mathsf{E}(u), m, \vartheta)\nabla m\Big) = \mathcal{S}(\mathsf{E}(u), m, \vartheta)e\cdot e + \mathbb{D}\mathsf{E}(\dot{u}){:}\mathsf{E}(\dot{u}) + \mathbb{D}_{\mathrm{H}}\nabla\mathsf{E}(\dot{u})\,\vdots\,\nabla\mathsf{E}(\dot{u}) + \alpha|\dot{m}|^2 + \mathcal{A}_1(\mathsf{E}(u), m, \vartheta){:}\mathsf{E}(\dot{u}) + \mathcal{A}_2(\mathsf{E}(u), m, \vartheta)\cdot\dot{m},$$

$$\mu_0(\dot{h} + \dot{m}) + \mathrm{curl}\,e = -\mu_0\nabla m\,\dot{u} - \mu_0(\mathrm{div}\,\dot{u})m,$$

$$\mathrm{curl}\,h - \mathcal{S}(\mathsf{E}(u), m, \vartheta)e = 0,$$

Pinning effects: phase field $\chi = \chi(\mathsf{E}(u), m)$ and additional dissipation $\zeta(\dot{\chi})$. Thus, in terms of the 6-tuple $(u, m, \vartheta, e, h, \omega)$, the original system expands to the following six equations/inclusion:

$$\varrho\,\ddot{u} - \mathrm{div}\Big(\sigma_{\mathsf{E}}(\mathsf{E}(u), m, \vartheta) + \mathbb{D}\mathsf{E}(\dot{u}) + \chi'_{\mathsf{E}}(\mathsf{E}(u), m)^\top \omega - \mathrm{div}\big(\mathbb{C}_{\rm H}\nabla\mathsf{E}(u) + \mathbb{D}_{\rm H}\nabla\mathsf{E}(\dot{u})\big)\Big) = f_0 - \mu_0 \nabla h^\top m,$$

$$\alpha\dot{m} - \frac{m\times\dot{m}}{\gamma(|m|)} - \lambda\,\Delta m + \sigma_m(\mathsf{E}(u), m, \vartheta) = p_0 + \mu_0 h - \chi'_m(\mathsf{E}(u), m)^\top \omega,$$

$$\dot{\vartheta} - \mathrm{div}\Big(\mathcal{K}_0(\mathsf{E}(u), m, \vartheta)\nabla\vartheta + \mathcal{K}_1(\mathsf{E}(u), m, \vartheta)\nabla\mathsf{E}(u) + \mathcal{K}_2(\mathsf{E}(u), m, \vartheta)\nabla m\Big)$$
$$= \mathcal{S}(\mathsf{E}(u), m, \vartheta)e\cdot e + \mathbb{D}\mathsf{E}(\dot{u}){:}\mathsf{E}(\dot{u}) + \mathbb{D}_{\rm H}\nabla\mathsf{E}(\dot{u})\,\vdots\,\nabla\mathsf{E}(\dot{u}) + \alpha|\dot{m}|^2 + \mathcal{A}_1(\mathsf{E}(u), m, \vartheta){:}\mathsf{E}(\dot{u}) + \mathcal{A}_2(\mathsf{E}(u), m, \vartheta)\cdot\dot{m} + \zeta\big(\chi'_{\mathsf{E}}(\mathsf{E}(u), m)\mathsf{E}(\dot{u}) + \chi'_m(\mathsf{E}(u), m)\dot{m}\big),$$

$$\mu_0(\dot{h} + \dot{m}) + \mathrm{curl}\, e = -\mu_0 \nabla m\,\dot{u} - \mu_0(\mathrm{div}\,\dot{u})m,$$

$$\mathrm{curl}\, h - \mathcal{S}(\mathsf{E}(u), m, \vartheta)e = 0,$$

$$\omega \in \partial\zeta\big(\chi'_{\mathsf{E}}(\mathsf{E}(u), m)\mathsf{E}(\dot{u}) + \chi'_m(\mathsf{E}(u), m)\dot{m}\big).$$

Some references:

- M.Arndt, M.Griebel, V. Novák, T. Roubíček, P.Šittner: Martensitic transformation in NiMnGa single crystals: numerical simulations and experiments. *Int. J. Plasticity* **22** (2006), 1943-1961.
- P. Plecháč, T. Roubíček: Visco-elasto-plastic model for martensitic phase transformation in shape-memory alloys. *M2AS* **25** (2002), 1281–1298.
- P. Podio-Guidugli, T. Roubíček, G. Tomassetti: A thermodynamically-consistent theory of the ferro/paramagnetic transition. *Archive Rat. Mech. Anal.* **198** (2010), 1057-1094.
- K.R. Rajagopal, T. Roubíček: On the effect of dissipation in shape-memory alloys. *Nonlinear Anal., Real World Appl.* **4** (2003), 581–597.
- T. Roubíček: Nonlinearly coupled thermo-visco-elasticity. *NoDEA*, submitted.
- T. Roubíček, G. Tomassetti: Thermodynamics of shape-memory alloys under electric current. *Zeit. angew. Math. Phys.* **61** (2010), 1-20.
- T. Roubíček, G. Tomassetti: Ferromagnets with eddy currents and pinning effects: their thermodynamics and analysis. *M3AS* **21** (2011), 29-55.
- T. Roubíček, G. Tomassetti: Phase transformations in electrically conductive ferromagnetic shape-memory alloys, their thermodynamics and analysis. *Archive Rat. Mech. Anal.*, submitted.
- T. Roubíček, G. Tomassetti, C. Zanini: The Gilbert equation with dry-friction-type damping. *J. Math. Anal. Appl.*, **355** (2009), 453–468.

Some preprints available on:

http://www.karlin.mff.cuni.cz/~roubicek/trpublic.htm

Some references:

- M.Arndt, M.Griebel, V. Novák, T. Roubíček, P.Šittner: Martensitic transformation in NiMnGa single crystals: numerical simulations and experiments. *Int. J. Plasticity* **22** (2006), 1943-1961.
- P. Plecháč, T. Roubíček: Visco-elasto-plastic model for martensitic phase transformation in shape-memory alloys. *M2AS* **25** (2002), 1281–1298.
- P. Podio-Guidugli, T. Roubíček, G. Tomassetti: A thermodynamically-consistent theory of the ferro/paramagnetic transition. *Archive Rat. Mech. Anal.* **198** (2010), 1057-1094.
- K.R. Rajagopal, T. Roubíček: On the effect of dissipation in shape-memory alloys. *Nonlinear Anal., Real World Appl.* **4** (2003), 581–597.
- T. Roubíček: Nonlinearly coupled thermo-visco-elasticity. *NoDEA*, submitted.
- T. Roubíček, G. Tomassetti: Thermodynamics of shape-memory alloys under electric current. *Zeit. angew. Math. Phys.* **61** (2010), 1-20.
- T. Roubíček, G. Tomassetti: Ferromagnets with eddy currents and pinning effects: their thermodynamics and analysis. *M3AS* **21** (2011), 29-55.
- T. Roubíček, G. Tomassetti: Phase transformations in electrically conductive ferromagnetic shape-memory alloys, their thermodynamics and analysis. *Archive Rat. Mech. Anal.*, submitted.
- T. Roubíček, G. Tomassetti, C. Zanini: The Gilbert equation with dry-friction-type damping. *J. Math. Anal. Appl.*, **355** (2009), 453–468.

Thanks a lot for your attention.

Some preprints available on:

http://www.karlin.mff.cuni.cz/~roubicek/trpublic.htm