Homework # 1 - Complex representations of dihedral groups

For $n \geq 3$ we denote D_{2n} the group of symmetries of n-sided regular polygon (note that $|D_{2n}| = 2n$, in the literature you can find also a different notation).

Looking at the action of D_{2n} on the set of vertices of the regular n-gon we can describe this group as the subgroup of the symmetric group $S(\{0, 1, 2, \ldots, n-1\})$ with underlying set $\{r_0, r_1, \ldots, r_{n-1}, s_0, s_1, \cdots, s_{n-1}\}$, where

$$r_i \colon x \mapsto x + i \bmod n; i \in \{0, 1, \ldots, n-1\}$$

$$s_i \colon x \mapsto i - x \bmod n; i \in \{0, 1, \ldots, n-1\}$$

This group can be also described using generators and relations as

$$D_{2n} = \langle x, y \mid x^n = 1, y^2 = 1, yxy = x^{-1} \rangle .$$

1. Find conjugacy classes in D_{2n} and deduce how many (up to equivalence) different irreducible representations of D_{2n} over $\mathbb{C}$ exists.
2. Find $[D_{2n}, D_{2n}]$ and use it to describe all degree one complex representations of D_{2n}.
3. Find all irreducible complex representations of D_{2n} up to equivalence and compute their characters. Don't forget to verify that these representations are indeed irreducible.
4. Prove or disprove $\mathbb{Q}D_8 \simeq \mathbb{Q} \times \mathbb{Q} \times \mathbb{Q} \times \mathbb{Q} \times \mathrm{M}_2(\mathbb{Q})$ as $\mathbb{Q}$-algebras.