1 1. způsob: $f(x) = g(x^2)\,2x = g(\varphi(x))\,\varphi'(x)$

kde $\varphi(x) = x^2$

$g(y) = \dfrac{1}{y^2+2y+5}$

1. VoS → $\int f(x)\,dx = G(x^2)$, $x \in \mathbb{R}$

kde $G(y) = \int g(y)\,dy = \frac{1}{4}\int \dfrac{dy}{\left(\frac{y+1}{2}\right)^2+1} = \frac{1}{2}\operatorname{arctg}\left(\frac{y+1}{2}\right)$

$y \in \mathbb{R}$

tedy: $\displaystyle\int_{-\infty}^{0} f(x)\,dx = \left[\frac{1}{2}\operatorname{arctg}\left(\frac{x^2+1}{2}\right)\right]_{-\infty}^{0} = \frac{1}{2}\operatorname{arctg}\frac{1}{2} - \frac{\pi}{4}$

$\doteq -0{,}55\ldots$

2. způsob: substituce $x = -\sqrt{t}$, $t \in (0, \infty)$

tj. $\varphi'(t) = -\dfrac{1}{2\sqrt{t}}\,dt$

pozor: $\displaystyle\int_{-\infty}^{0} f(x)\,dx = \int_{0}^{+\infty} f(\varphi(t))\,|\varphi'(t)|\,dt$

$\displaystyle = - \int_{0}^{\infty} \frac{dt}{t^2+2t^2+1}$

② $f(x) = \underbrace{\frac{x-\frac{1}{2}}{x+\frac{1}{2}}}_{h(x)} \cdot \frac{\cos x}{\sqrt{x}}$

i) pomocná úloha $\quad \tilde{I} = \int_0^{+\infty} \frac{\cos x}{\sqrt{x}} = \int_0^1 \cdots + \int_1^{+\infty} \cdots = \tilde{I}_1 + \tilde{I}_2$

$\tilde{I}_1$ konv. ... $\left|\frac{\cos x}{\sqrt{x}}\right| \leq \frac{1}{\sqrt{x}}$, $\int_0^1 \frac{dx}{\sqrt{x}} < +\infty$

$\tilde{I}_2$ konv. ... $\frac{\cos x}{\sqrt{x}}$ spojitá na $[1, +\infty)$,

$\int \cos x = -\sin x$ ... omezená

$\frac{1}{\sqrt{x}} \to 0,\ x \to +\infty$, klesá

ii) $h(x) = 1 - \frac{1}{x+\frac{1}{2}}$ ... omezená, monotónní v $(0, +\infty)$

$\Rightarrow \int_0^{+\infty} f(x)\,dx$ konv. $\quad$ (Dirichlet na $(0,1]$ a $[1,+\infty)$)

③ $V = \frac{\pi}{4} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos^2 x\,dx = 2 \cdot \frac{\pi}{4} \cdot \frac{1}{2} \int_0^{\frac{\pi}{4}} 1 - \cos 2x\,dx$

$= \frac{1}{16}\pi(\pi+2) \doteq 1{,}01$