(1) $\displaystyle\int_0^{\ln 2} \sqrt{e^x-1}\,dx \quad \left|\begin{array}{l} e^x-1 = u \in (0,1) \\ x = \ln(1+u) \\ dx = \frac{du}{1+u} \end{array}\right| = \int_0^1 \frac{\sqrt{u}}{1+u}\,du$

$\left.\begin{array}{l} \sqrt{u} = v \in (0,1) \\ u = v^2 \\ du = 2v\,dv \end{array}\right| = \displaystyle\int_0^1 \frac{2v^2}{1+v^2}\,dv = 2\int_0^1 1 - \frac{1}{1+v^2}\,dv$

$$= 2\left[v - \operatorname{arctg} v\right]_0^1 = 2 - \frac{\pi}{2}$$

Pozn: lze spojit do jedné substituce $\sqrt{e^x-1} = t$.

(2) $\displaystyle\int_2^{+\infty} \frac{dx}{x^2} = \left[-\frac{1}{x}\right]_2^{+\infty} = -\frac{1}{+\infty} - \left(-\frac{1}{2}\right) = \frac{1}{2}$

(8) $\displaystyle\int_0^1 x^\lambda \ln x\,dx = \underbrace{\left[\frac{x^{\lambda+1}}{\lambda+1}\ln x\right]_0^1}_{=0} - \int_0^1 \frac{x^\lambda}{\lambda+1}\,dx = \left[-\frac{x^{\lambda+1}}{(\lambda+1)^2}\right]_0^1 = \frac{-1}{(\lambda+1)^2}$

$u' = x^\lambda,\ u = \frac{x^{\lambda+1}}{\lambda+1}$

$v = \ln x,\ v' = \frac{1}{x}$

0, neboť $\lambda+1>0$; tj:

$x^{\lambda+1}\cdot\ln x \to 0,\ x\to 0+$

(10) $\displaystyle\int_0^{\frac{\pi}{2}} \underbrace{\operatorname{tg} x}_{=\frac{\sin x}{\cos x} = -(\ln(\cos x))'}\,dx = \left[-\ln(\cos x)\right]_0^{\frac{\pi}{2}} = -(-\infty) - \ln 1 = +\infty,$

$x \in (0, \frac{\pi}{2})$.

neboť $x \to \frac{\pi}{2} \Rightarrow \cos x \to 0$

$\cos x > 0$

$\Rightarrow \ln(\cos x) \to -\infty$

Varianta: substituce

$\left.\begin{array}{l} \cos x = t \in (0,1) \\ -\sin x\,dx = dt \end{array}\right| = \displaystyle\int_0^1 \frac{dt}{t} = \left[\ln t\right]_0^1 = 0 - (-\infty)$

$= +\infty.$

4.1
$$\int_{-\infty}^{0} \frac{e^{4x}+4e^{3x}-e^{2x}-2e^{x}}{(e^{2x}+1)(2e^{2x}+3e^{x}+1)}\,dx \quad \Bigg| \quad \begin{aligned} e^x &= y \in (0,1) \\ x &= \ln y \\ dx &= \frac{dy}{y} \end{aligned}$$

$$= \underbrace{\int_0^1 \frac{y^3+4y^2-y-2}{(y^2+1)(2y^2+3y+1)}\,dy}_{g(y)} \quad ; \quad \text{jmenovatel: } 2y^2+3y+1 = (2y+1)(y+1)$$

rozklad: $g(y) = \frac{Ay+B}{y^2+1} + \frac{C}{2y+1} + \frac{D}{y+1}$; dosazení: $A=2,\ B=0$, $C=-1,\ D=-1$

$$\int g(y) = \int \frac{2y}{y^2+1} - \frac{1}{2y+1} - \frac{1}{y+1}\,dy = \ln(y^2+1) - \frac{1}{2}\ln|2y+1| - \ln|y+1| =: G(y)$$

$$I = \left[G(y)\right]_0^1 = -\frac{1}{2}\ln 3.$$

$y \in (-\infty,-1)$, $(-1,-\frac{1}{2})$, $(-\frac{1}{2},+\infty)$.

$$G(y) = \ln \frac{y^2+1}{\sqrt{|2y+1|}\,|y+1|}$$

4.2
$$\int_0^1 \frac{dx}{1+\sqrt{\frac{x+1}{x}}} \quad \Bigg| \quad \begin{aligned} t &= \sqrt{\frac{x+1}{x}} \in (\sqrt{2},+\infty) \\ x &= \frac{1}{t^2-1}\ ;\ dx = \frac{-2t}{(t^2+1)^2} \end{aligned} \quad \Bigg| = \underbrace{\int_{\sqrt{2}}^{+\infty} \frac{|-2t|}{(t+1)^3(t-1)^2}\,dt}_{g(t)}$$

$(t>0)$

$$g(t) = \frac{A}{t+1} + \frac{B}{(t+1)^2} + \frac{C}{(t+1)^3} + \frac{D}{t-1} + \frac{E}{(t-1)^2},$$

$(t>0)$

dosazením $A=-D=\frac{1}{8}$, $B=0,\ C=-\frac{1}{2},\ E=\frac{1}{4}$

$$\int g(t)\,dt = \frac{1}{8}\ln\left|\frac{t+1}{t-1}\right| + \frac{1}{4}\cdot\frac{1}{(t+1)^2} - \frac{1}{4}\cdot\frac{1}{t-1} = G(t)$$

$$I = \left[G(t)\right]_{\sqrt{2}}^{+\infty} = 0 - G(\sqrt{2}) \doteq 0.340\ldots$$