čt. 19.2.

(i) $$\frac{e^x - \sin x - 1}{x^2} = \frac{1}{x^2}\Big(\underbrace{1 + x + \frac{x^2}{2} + o(x^2)}_{\exp(x)} - \underbrace{\left(x + o(x^2)\right)}_{\sin x} - 1\Big)$$

$$= \frac{1}{x^2}\left(\frac{1}{2}x^2 + o(x^2)\right) = \frac{1}{2} + o(1) \to \frac{1}{2}$$

(ii) $$e^y = 1 + y + \frac{1}{2}y^2 + o(y^2)$$
$$e^{-\frac{x^2}{2}} = 1 - \frac{x^2}{2} + \frac{1}{8}x^4 + o(x^4)$$

čitatel $= -\frac{1}{12}x^4 + o(x^4)$; limita $= -\frac{1}{12}$

(iii) $$e^{\sin x} = 1 + \left(x - \frac{1}{6}x^3 + o(x^3)\right)$$
$$+ \frac{1}{2}\left(x - \frac{1}{6}x^3 + o(x^3)\right)^2$$
$$+ \frac{1}{6}\left(x - \frac{1}{6}x^3 + o(x^3)\right)^3$$
$$+ o\left(\left(x - \frac{1}{6}x^3 + o(x^3)\right)^3\right) = 1 + x + \frac{1}{2}x^2 + o(x^3)$$

čitatel $= \frac{1}{6}x^3 + o(x^3)$; limita $= \frac{1}{6}$

(iv) $$\cosh(x) = \frac{1}{2}(e^x - e^{-x}) = 1 + \frac{1}{2}x^2 + \underbrace{\frac{1}{24}x^4 + o(x^4)}_{o(x^3)}$$

čitatel: $-x^2 + o(x^3)$

celkem: $$\frac{-x^2 + o(x^3)}{x^3} = -\frac{1}{x} + o(x^3) \to -\infty,\ x \to 0+$$

st. 20.2.

(i) $\frac{1}{1-y} = 1+y+y^2+y^3+y^4+o(y^4)$

$$\frac{1}{1-(x-x^2)} = 1+(x-x^2)+(x-x^2)^2+\underbrace{(x-x^2)^3}_{x^3-3x^4+o(x^4)}+\underbrace{(x-x^2)^4}_{x^4+o(x^4)}+\underbrace{o((x-x^2)^4)}_{o(x^4)}$$

$$= 1+x-x^3-x^4+o(x^4)$$

celkem: $\underline{1+2x+2x^2-2x^4+o(x^4)}$

(ii) $f\left(\frac{1}{y}\right) = \frac{1}{y\sqrt{y}}\left(\sqrt{1+\frac{1}{y}}+\sqrt{\frac{1}{y}-1}-2\sqrt{\frac{1}{y}}\right)$

$$= \frac{1}{y^2}\left(\sqrt{1+y}+\sqrt{1-y}-2\right) \to -\frac{1}{4}$$

nelox. čitatel $= (1+y)^{1/2}+(1-y)^{1/2}-2 = -\frac{1}{4}y^2+o(y^2)$

dle vzorce $(1+y)^{1/2} = 1+\frac{1}{2}y-\frac{1}{8}y^2+o(y^2)$

(iii) $\cos(\operatorname{tg} x) = 1-\frac{1}{2}x^2-\frac{7}{24}x^4+o(x^4)$

$n=4$, $\lim = \frac{1}{24}-\left(-\frac{7}{24}\right) = \frac{1}{3}$.

(iv) $f\left(\frac{1}{y}\right) = \left(\frac{1}{y^3}-\frac{1}{y^2}+\frac{1}{2y}\right)e^y-\sqrt{\frac{1}{y^6}+1}$

$$= \frac{1}{y^3}\cdot\left[\underbrace{\left(1-y+\frac{1}{2}y^2\right)e^y}_{1+\frac{1}{6}y^3+o(y^3)}-\underbrace{(1+y^6)^{1/2}}_{1+o(y^3)}\right] \to \frac{1}{6}$$