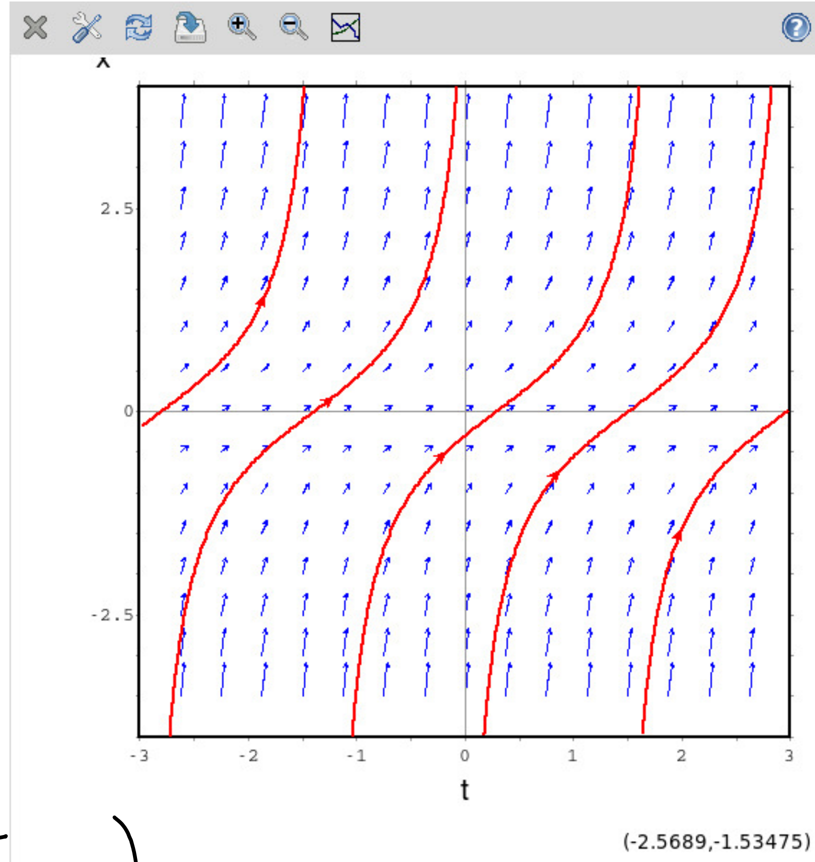
1) $$x' = x^2+1$$

$$\frac{dx}{x^2+1} = dt$$

$$\underbrace{\operatorname{arctg} x}_{\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)} = t+c$$



$$\Rightarrow \quad t+c \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$$

sj. $t \in I_c = \left(-\frac{\pi}{2}-c, \frac{\pi}{2}-c\right)$.

Solution: $\boxed{x(t) = \operatorname{tg}(t+c), \quad t \in I_c}$

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2) $$x' = e^x(t+1)$$

$$e^{-x}dx = (t+1)\,dt$$

$$\underbrace{-e^{-x}}_{\in(-\infty,0)} = \frac{1}{2}t^2+t+c$$

$$\Rightarrow \quad \frac{1}{2}t^2+t+c < 0$$

$$(t+1)^2 < 1-2c \quad \left(\text{hence } c < \frac{1}{2}\right)$$

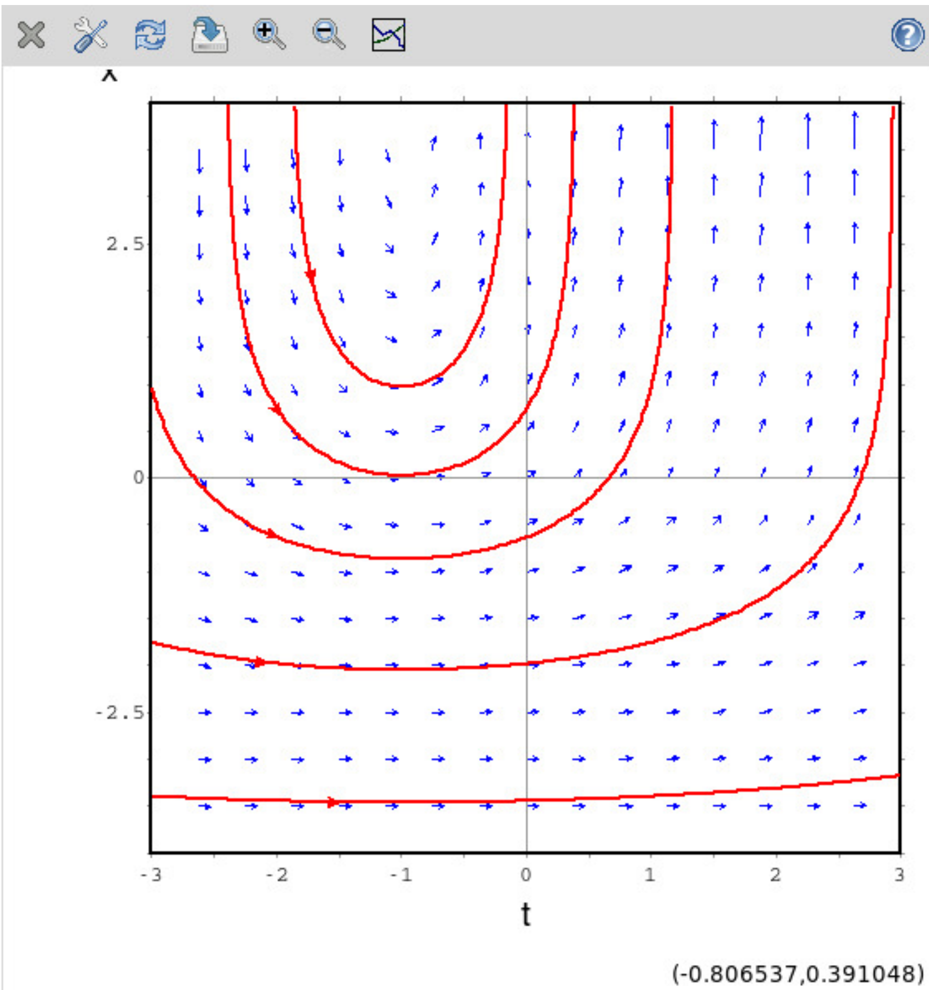
$$|t+1| < \sqrt{1-2c}$$

Solution:

$\mathcal{S}$: $t \in I_c := \left(-1-\sqrt{1-2c}, -1+\sqrt{1-2c}\right)$

$c < \frac{1}{2}$ fixed

$$x(t) = -\ln\left(-\frac{1}{2}t^2 - t - c\right)$$



3) $x' = \frac{1}{t}(x^2 - x) \Longrightarrow t \in (-\infty, 0)$ or $(0, +\infty)$

stationary solution:

$x \equiv 0$ or $x \equiv 1$

now consider: $x \in (-\infty, 0)$ or $(0, 1)$ or $(0, +\infty)$

$$\frac{dx}{x^2-x} = \frac{dt}{t}$$

recall: partial fractions

$$\frac{1}{x^2-x} = \frac{1}{x(x-1)} = \frac{1}{x-1} - \frac{1}{x}$$

$$\int \frac{dx}{x^2-x} = \ln|x-1| - \ln|x|$$

$$= \ln\left|\frac{x-1}{x}\right|$$

$$\Rightarrow \ln\left|\frac{x-1}{x}\right| = \ln|t| + c \qquad (c \in \mathbb{R})$$

$$\left|\frac{x-1}{x}\right| = e^c|t|$$

$$\frac{x-1}{x} = kt \qquad (k = \pm e^c \in \mathbb{R}\setminus\{0\})$$

$$\Rightarrow \boxed{x(t) = \frac{1}{1-kt}}$$

in every interval not containing $t=0$ or $t=\frac{1}{k}$

that means:

1) $k>0$: $t \in (-\infty, 0)$ or $(0, \frac{1}{k})$ or $(\frac{1}{k}, +\infty)$

2) $k<0$: $t \in (-\infty, \frac{1}{k})$ or $(\frac{1}{k}, 0)$ or $(0, +\infty)$