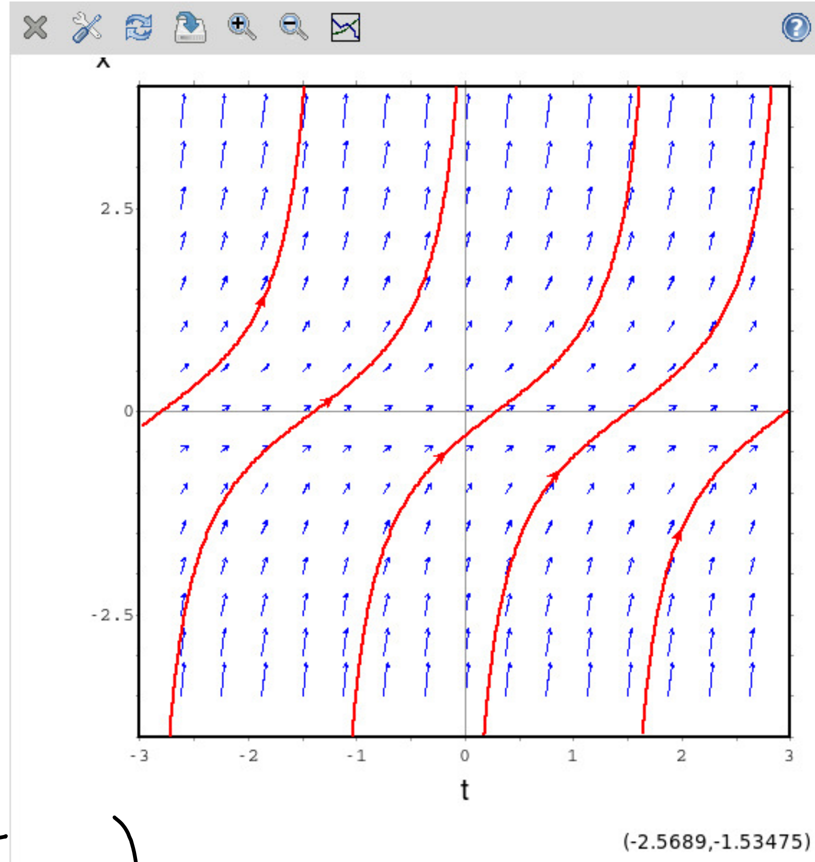
1) $x' = x^2+1$

$$\frac{dx}{x^2+1} = dt$$

$$\underbrace{\operatorname{arctg} x}_{\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)} = t+c$$

$\Rightarrow \quad t+c \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

tj. $t \in I_c = \left(-\frac{\pi}{2}-c, \frac{\pi}{2}-c\right)$.



Řešení: $\boxed{x(t) = \operatorname{tg}(t+c),\ t \in I_c}$

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2) $x' = e^x(t+1)$

$$e^{-x}\,dx = (t+1)\,dt$$

$$\underbrace{-e^{-x}}_{\in(-\infty,0)} = \frac{1}{2}t^2+t+c$$

$\Rightarrow \quad \frac{1}{2}t^2+t+c<0$

$(t+1)^2 < 1-2c$ (nutně $c<\frac{1}{2}$)

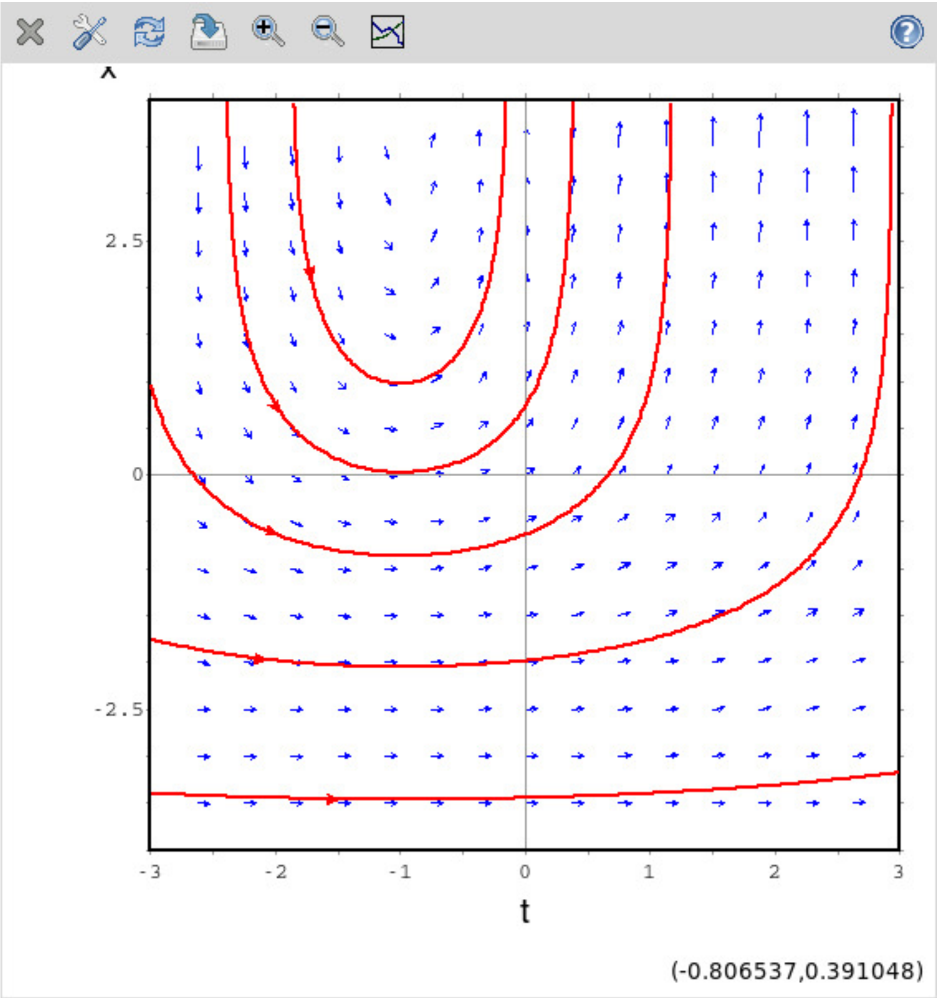
$|t+1| < \sqrt{1-2c}$

Řešení:

$$\text{tj. } t \in I_c := \left(-1-\sqrt{1-2c}, -1+\sqrt{1-2c}\right)$$

$c < \frac{1}{2}$ pevné

$$x(t) = -\ln\left(-\frac{1}{2}t^2 - t - c\right)$$



3) $x' = \frac{1}{t}(x^2 - x) \implies t \in (-\infty, 0)$ nebo $(0, +\infty)$

stacionární řešení:

$x \equiv 0$ nebo $x \equiv 1$

zbývá: $x \in (-\infty, 0)$ nebo $(0, 1)$ nebo $(0, +\infty)$

$$\frac{dx}{x^2-x} = \frac{dt}{t}$$

Opakování: parciální zlomky

$$\frac{1}{x^2-x} = \frac{1}{x(x-1)} = \frac{1}{x-1} - \frac{1}{x}$$

$$\int \frac{dx}{x^2-x} = \ln|x-1| - \ln|x|$$
$$= \ln\left|\frac{x-1}{x}\right|$$

$$\Rightarrow \ln\left|\frac{x-1}{x}\right| = \ln|t| + c \qquad (c \in \mathbb{R})$$

$$\left|\frac{x-1}{x}\right| = e^c |t|$$

$$\frac{x-1}{x} = kt \qquad (k = \pm e^c \in \mathbb{R} \setminus \{0\})$$

$$\Rightarrow \boxed{x(t) = \frac{1}{1-kt}}$$, ve všech intervalech neobsahujících $t=0$ ani $t=\frac{1}{k}$

1) $k>0$: $t \in (-\infty, 0)$ nebo $(0, \frac{1}{k})$ nebo $(\frac{1}{k}, +\infty)$

2) $k<0$: $t \in (-\infty, \frac{1}{k})$ nebo $(\frac{1}{k}, 0)$ nebo $(0, +\infty)$