Pozn. 1) $x' = Ax$, $\lambda \in \sigma(A)$, v ... vl. vektor

$\Rightarrow$ $x(t) = e^{\lambda t} v$ je řešení

Dz: $x' = \left(e^{\lambda t} v\right)' = \lambda e^{\lambda t} v$ ... LS

$Ax = A\left(e^{\lambda t} v\right) = e^{\lambda t} \underbrace{Av}_{= \lambda v}$

2) nechť $A, \tilde{A}$ jsou podobné (tj. $\tilde{A} = CAC^{-1}$, značíme $A \sim \tilde{A}$)

matice přechodu

$\boxed{x' = Ax}$, řešení $x(t)$

označ $y(t) = C x(t)$ $\left(\Leftrightarrow x(t) = C^{-1} y(t)\right)$

$y' = Cx' = CAx = CAC^{-1} y$

$\Rightarrow$ $\boxed{y' = \tilde{A} y}$

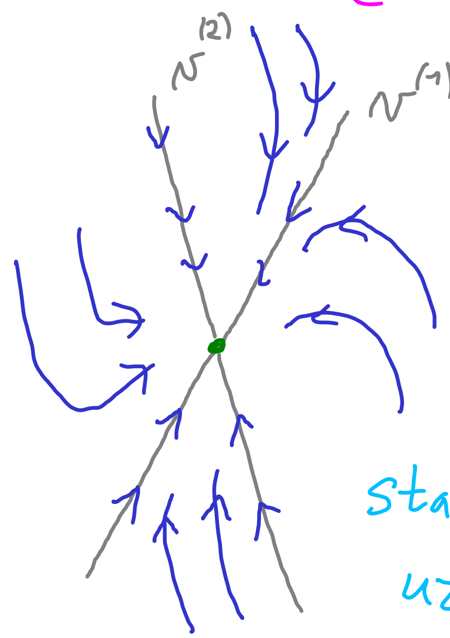
Řce $x' = Ax$, $A \in \mathbb{R}^{2\times 2}$ – klasifikace

1) $\sigma(A)$: $\lambda_2 < \lambda_1 < 0$;

tj. $A \sim \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$

nechť: $v^{(1)}, v^{(2)}$

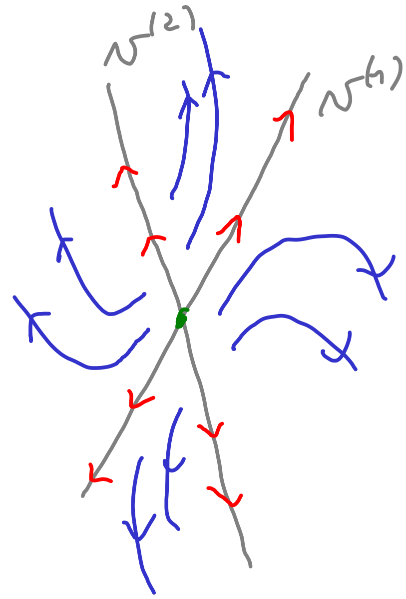
$\Rightarrow$ řešení $e^{\lambda_1 t} v^{(1)}$, $e^{\lambda_2 t} v^{(2)}$



stabilní uzel

2) $\sigma(A)$: $0 < \lambda_1 < \lambda_2$

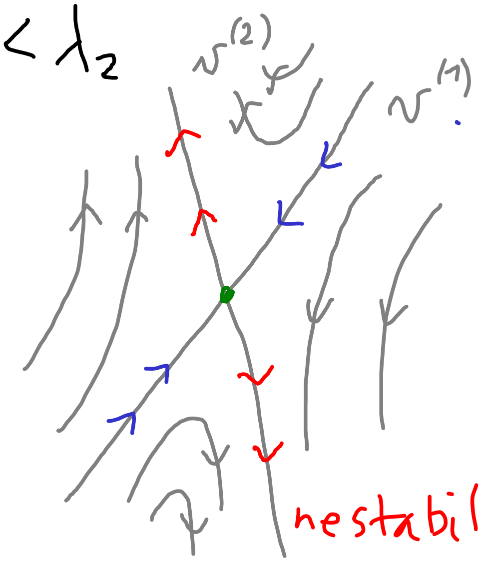
$v^{(1)}, v^{(2)}$



nestabilní uzel

3) $\sigma(A)$: $\lambda_1 < 0 < \lambda_2$

hyperbolický bod

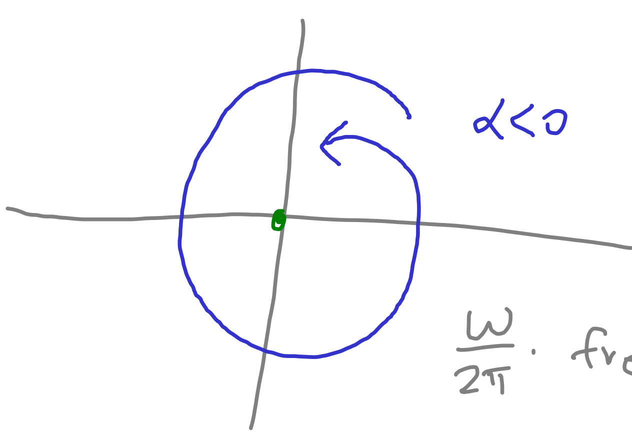


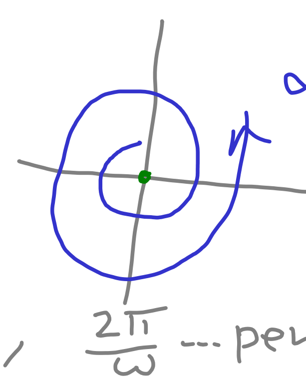
$v^{(1)}$ ... stabilní směr

nestabilní směr

4) $\sigma(A)$: $\alpha \pm i\omega$ $\quad (\alpha, \omega \in \mathbb{R}, \omega \neq 0)$

$$A \sim \begin{pmatrix} \alpha, & -\omega \\ \omega, & \alpha \end{pmatrix} \; ; \quad \text{řešení:} \quad e^{\alpha t}\begin{pmatrix} \cos\omega t \\ \sin\omega t \end{pmatrix}, \; e^{\alpha t}\begin{pmatrix} -\sin\omega t \\ \cos\omega t \end{pmatrix}$$





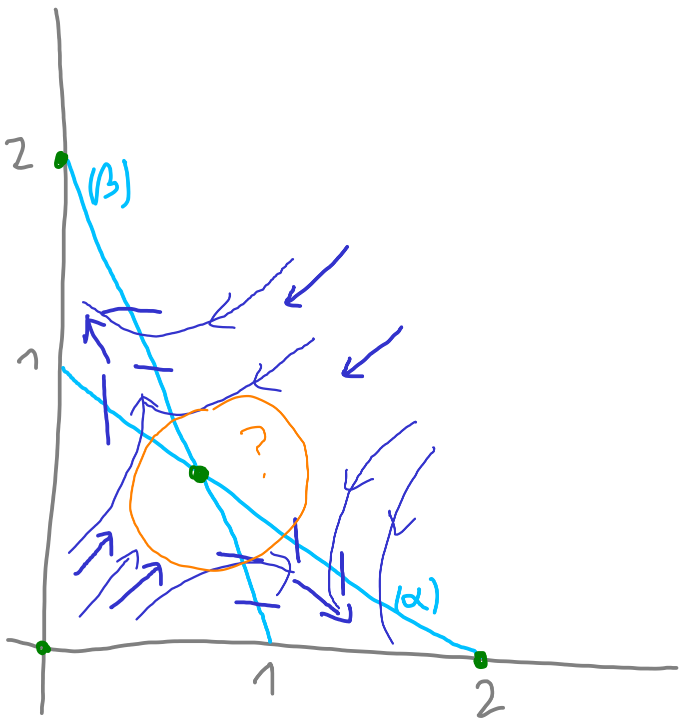
$\frac{\omega}{2\pi}$: frekvence, $\frac{2\pi}{\omega}$ ... perioda

Př. model konkurence (d.ú.)

$$x' = r\left(1 - \frac{x + ay}{K}\right)x$$
$$y' = s\left(1 - \frac{y + bx}{L}\right)y$$

konkrétně: $x' = \left(1 - \frac{x}{2} - y\right)x$

$y' = (2 - 2x - y)y$



$x' = 0$: $x = 0$ nebo

$y = 1 - \frac{x}{2}$ (α)

$y' = 0$: $y = 0$ nebo

$y = 2 - 2x$ (β)

stac. body: $[0,0]$, $[2,0]$

$[0,2]$, $\left[\frac{2}{3}, \frac{2}{3}\right]$

průběhy řešení:

$x' \gtrless 0 \iff$ nad (pod) (α)

$y' \gtrless 0 \iff$ vpravo (vlevo) od (β)

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linearizace v okolí $\left[\frac{2}{3}, \frac{2}{3}\right]$: $X = F(X)$; $X = \begin{pmatrix} x \\ y \end{pmatrix}$

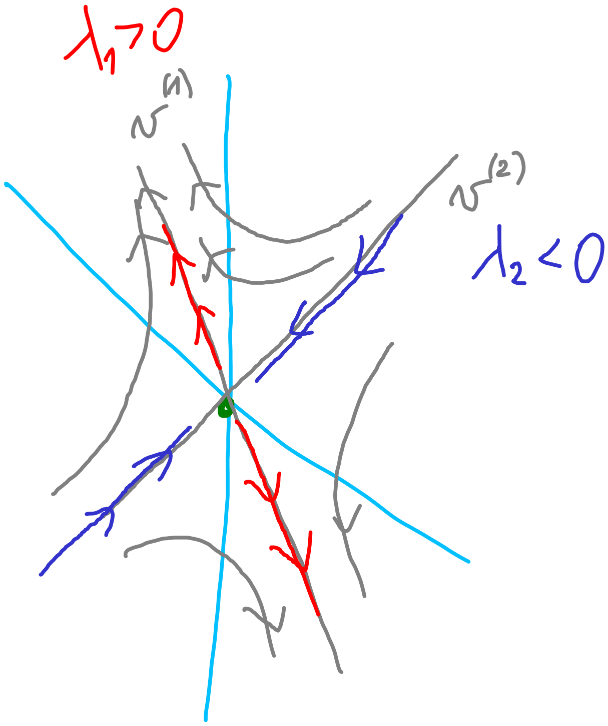
$$F = \begin{pmatrix} x - \frac{x^2}{2} - xy \\ 2y - 2xy - y^2 \end{pmatrix}$$

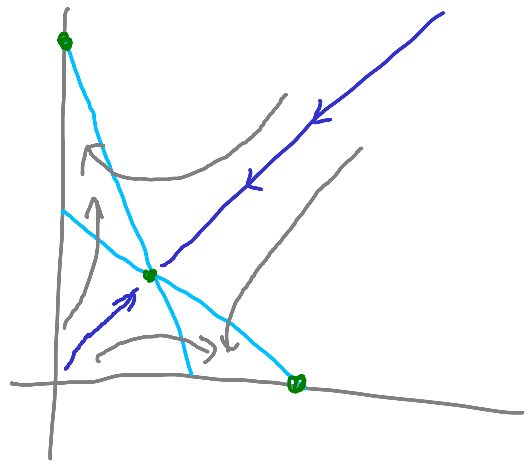
$$DF = \begin{pmatrix} \frac{\partial F_1}{\partial x}, & \frac{\partial F_2}{\partial x} \\ \frac{\partial F_2}{\partial x}, & \frac{\partial F_2}{\partial y} \end{pmatrix} = \begin{pmatrix} 1-x-y, & -x \\ -2y, & 2-2x-2y \end{pmatrix}$$

$$A = DF\begin{pmatrix} 2/3 \\ 2/3 \end{pmatrix} = \begin{pmatrix} -\frac{1}{3}, & -\frac{2}{3} \\ -\frac{4}{3}, & -\frac{2}{3} \end{pmatrix} \Rightarrow \lambda_{1,2} = \frac{-3 \pm \sqrt{33}}{2}$$

$$v^{(1)} = \left(1, \frac{+1-\sqrt{33}}{2}\right)$$

$$v^{(2)} = \left(1, \frac{1+\sqrt{33}}{2}\right)$$

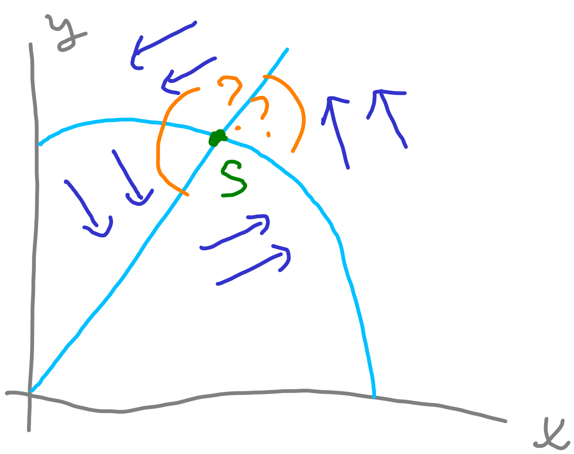




Př.: Holling-Tanner:

$$x' = \left(r\left(1-\frac{x}{K}\right) - \frac{my}{a+x}\right)x$$

$$y' = s\left(1 - \frac{Py}{x}\right)y$$



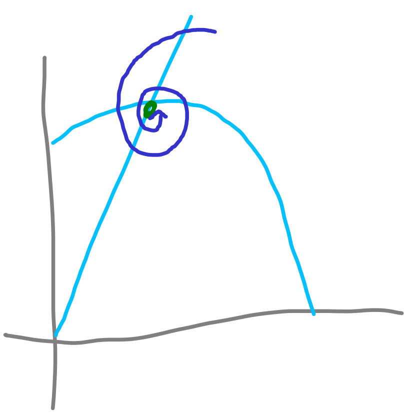
Konkrétně :

$$\left.\begin{aligned} x' &= \left(1 - x - \frac{y}{1/4 + x}\right)x \\ y' &= \left(1 - \frac{4y}{3x}\right)y \end{aligned}\right\} X' = F(X)$$

$\Rightarrow \quad S = [1/2, 3/8] \quad \cdots$ linearizace ?

$$X = \begin{pmatrix} x \\ y \end{pmatrix}, \quad F = \begin{pmatrix} x - x^2 - \dfrac{xy}{1/4 + x} \\ y - \dfrac{4y^2}{3x} \end{pmatrix}$$

$$DF = \begin{pmatrix} 1 - 2x - \dfrac{y}{4(1/4 + x)^2}, & \dfrac{-x}{1/4 + x} \\ \dfrac{4y^2}{3x^2}, & 1 - \dfrac{8y}{3x} \end{pmatrix}; \quad A = DF\begin{pmatrix} 1/2 \\ 3/8 \end{pmatrix} = \begin{pmatrix} -1/6, & -2/3 \\ 3/4, & -1 \end{pmatrix}$$



$$\Rightarrow \quad \lambda = \frac{-7 \pm i\sqrt{47}}{12}$$