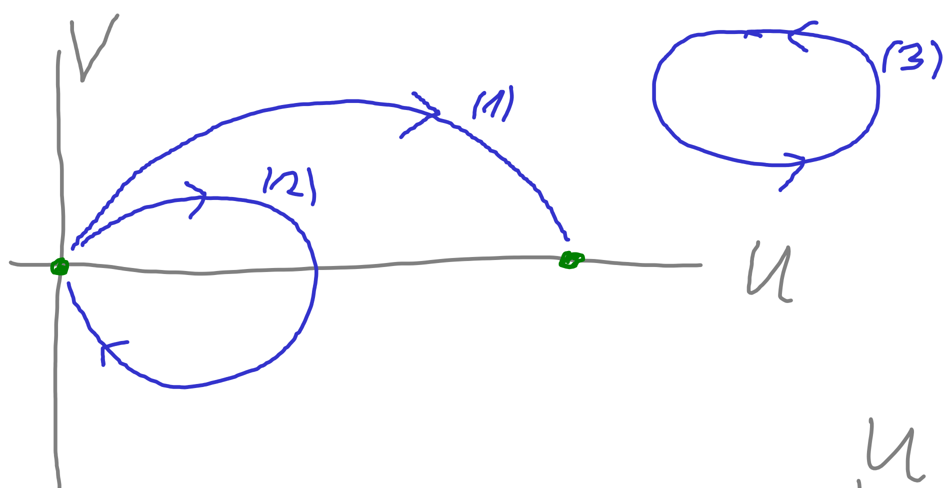
Problém: $\partial_t u = F(u, \partial_x u) + \varepsilon \partial_{xx} u$ $\quad (u = u(t,x))$

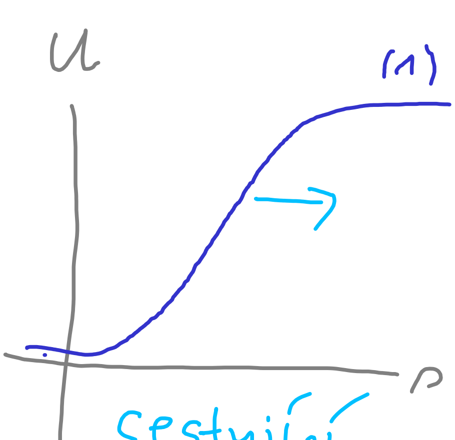
Ansatz: $u(t,x) = \underbrace{U}_{\text{profil}}(x - \underbrace{c}_{\text{rychlost}}t)$, $U = U(\rho)$

$$\Rightarrow \quad -cU' = F(U, -cU') + \varepsilon U''$$

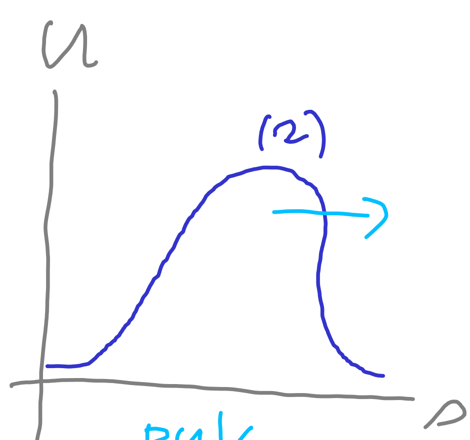
$$\Leftrightarrow \quad \boxed{U' = V, \quad \varepsilon V' = -cV - F(U, -cV)}$$

Pozn.

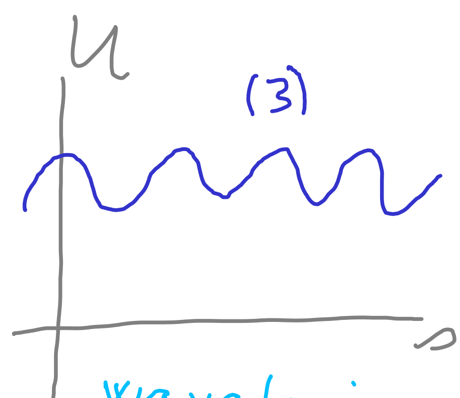




cestující vlny



puls (soliton)



wave train (vlnovlak)

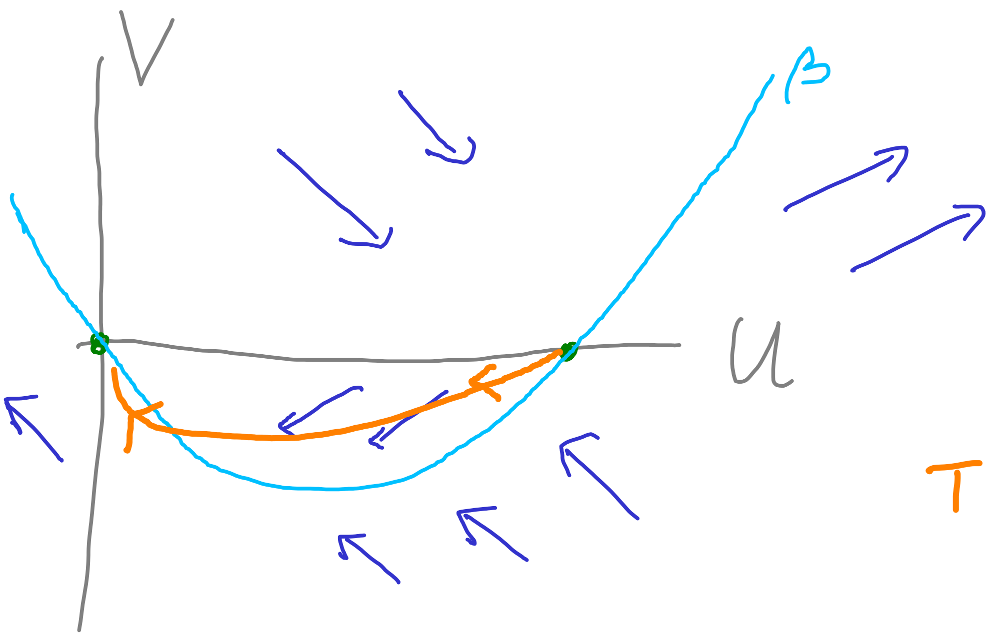
Př.: Fisherova rce : $\partial_t u = r\left(1-\frac{u}{K}\right)u + \varkappa \partial_{xx} u$

hecht: $u(t,x) = U(x-ct)$, BÚNO: $c>0$

$r=K=\varkappa=1$

$\Rightarrow \quad -cU' = (1-U)U + U''$

$\Leftrightarrow \quad \begin{bmatrix} U' = V \\ V' = U(U-1) - cV \end{bmatrix}$



TIP: tvar orbity (vlny)

$U' = 0 \Leftrightarrow V = 0$

$V' = 0 \Leftrightarrow V = \frac{1}{c} U(U-1)$ (B)

Věta 4 $X' = F(X)$; $F(X_0) = 0$

$F \in C^1(\mathcal{U}(X_0))$

Nechť: $\exists \lambda \in \sigma(A)$, $A = DF(X_0)$,

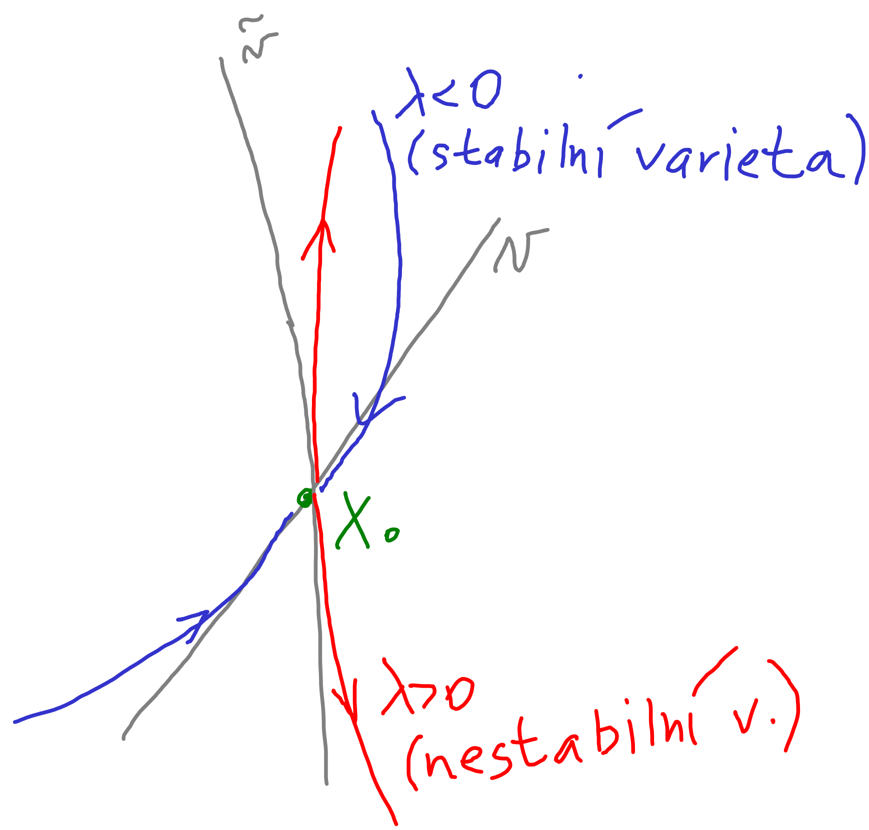
$\lambda < 0$, jednoduché

$(\lambda > 0)$

Pak: $\exists$ řešení (v okolí X_0, ± jediné)

t. ž. $X(t) \sim X_0 + v \cdot e^{\lambda t}$ $(t \to -\infty)$ $t \to \infty$

vl. vektor



$$u' = v$$
$$v' = u(u-1) - cv$$

$X = \begin{pmatrix} u \\ v \end{pmatrix}$

$F(X) = \begin{pmatrix} v \\ u^2 - u - cv \end{pmatrix}$

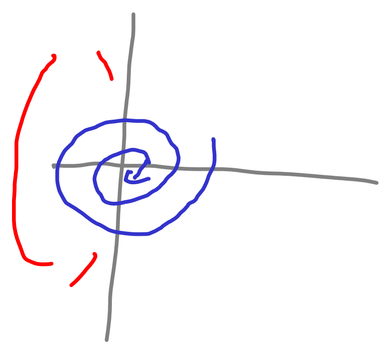
$\nabla F = \begin{pmatrix} 0, & 1 \\ 2u-1, & -c \end{pmatrix}$

stac. body: $X_0 = [0,0]$: $A_0 = \begin{pmatrix} 0, & 1 \\ -1, & c \end{pmatrix}$

$\lambda_{1,2} = \frac{1}{2}\left(-c \pm \sqrt{c^2-4}\right)$

$\Rightarrow$ nutně: $\boxed{c > 2}$ ... jinak: $c^2 - 4 < 0$

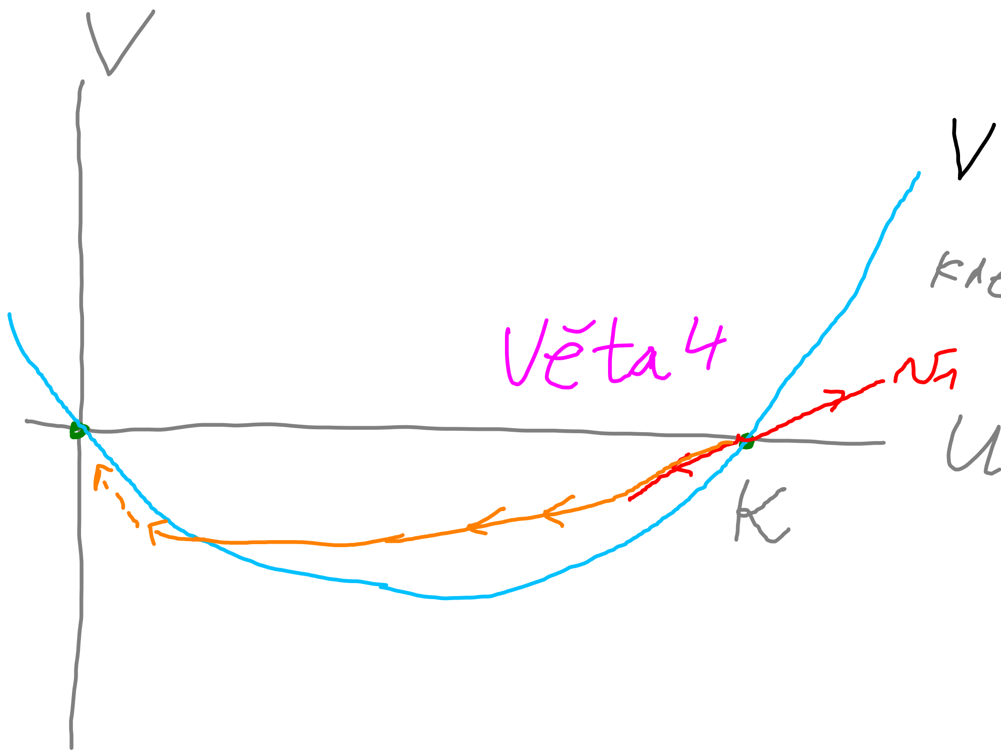
$\Rightarrow \lambda_1 = \alpha \pm i\beta$



$\Rightarrow u < 0$, nechci

Pozn: $\lambda_{1,2} < 0$, reálné $\Rightarrow$ $[0,0]$ asympt. stabilní bod $[1,0]$: $A_1 = \begin{pmatrix} 0, 1 \\ 1, -c \end{pmatrix} \Rightarrow \lambda_{1,2} = \dfrac{-c \pm \sqrt{c^2+4}}{2}$

spec: $\lambda_1 = \dfrac{-c+\sqrt{c^2+4}}{2} > 0, \quad v_1 = (1, \lambda_1)$



$V = \beta(U)$

kde $\beta(U) = \frac{1}{c} U(U-1)$

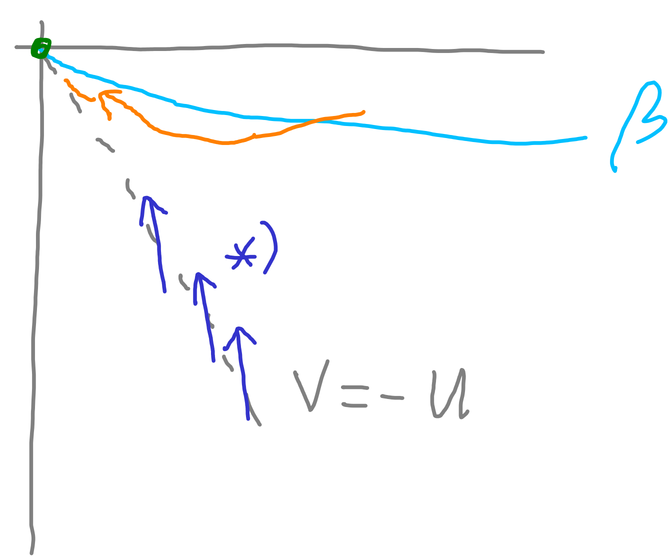
tvrdíme (lehce se spočte): $\beta'(1) > \lambda_1$

zbývá: $\rightarrow (0,0)$

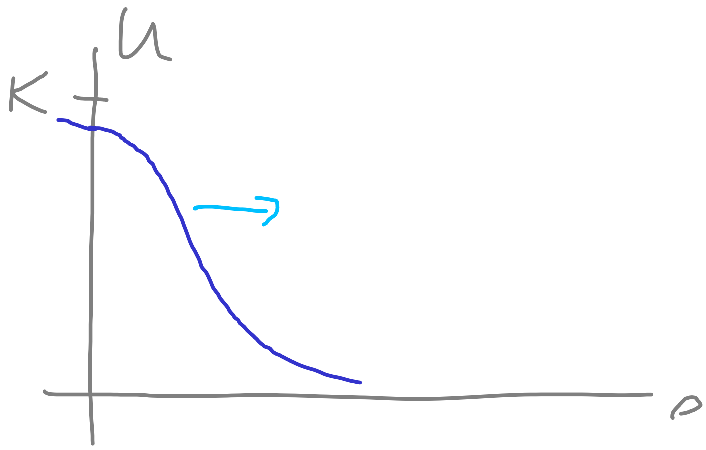
$\beta'(0) = -\frac{1}{c} > -1$

*) $\dfrac{dV}{dU} = \dfrac{V'}{U'} = \dfrac{-cV + U(U-1)}{V}$

$= -c+1-U \leq 1-c < -1$
$(c>2)$



Závěr: $\forall c > 2 \ \exists!$ cest. vlna



Příklad. neuronové pole:

$$\partial_t u = a u + b \partial_{xx} u, \qquad a, b \in \mathbb{R}$$

$u = u(t, x)$

$$u(t,x) = U(x - ct) \Rightarrow -cU' = aU + bU''$$

$$bU'' + cU' + aU = 0$$

(lin. rce s konst. koef.)

char. pol. $b\lambda^2 + c\lambda + a = 0$

$$\lambda_{1,2} = \frac{-c \pm \sqrt{c^2 - 4ab}}{2}$$

chci: omezená řeš.

tj.: $\lambda = \pm i\omega \Rightarrow c = 0, \ ab > 0$

$\omega \neq 0$

modifikace: zpoždění $\tau > 0$ (pevné)

$$\partial_t u(t,x) = a\, u(t-\tau, x) + b\, \partial_{xx} u(t-\tau, x)$$

nechť $u(t,x) = U(x-c,t)$ $\quad s = x-ct,\ \sigma = c\tau$

$$\Rightarrow\ -c\,U'(x-ct) = a\,U(x-c(t-\tau)) + b\,U''(x-c(t-\tau))$$

$$b\,U''(s+\sigma) + c\,U'(s) + a\,U(s+\sigma) = 0$$

$$b\,U''(s) + c\,U'(s-\sigma) + a\,U(s) = 0$$

Ansatz: $U(s) = e^{\lambda s},\ \lambda \in \mathbb{C}$

$$\Rightarrow\ b\lambda^2 \cancel{e^{\lambda s}} + c\lambda e^{\lambda(\cancel{s}-\sigma)} + a\cancel{e^{\lambda s}} = 0$$

char. rce: $b\lambda^2 + c\lambda e^{-\sigma\lambda} + a = 0$

cíl: $\exists$ netriv. řeš.: $\lambda = i\omega,\ \omega \neq 0$

dosaď:

$$b\lambda^2 + c\lambda e^{-\sigma\lambda} + a = 0 \qquad \underline{\lambda = i\omega}$$

$$\Rightarrow \quad b(i\omega)^2 + c\,\underline{i\omega} \cdot \underbrace{e^{-i\sigma\omega}}_{= \cos\sigma\omega - i\,\underline{\sin\sigma\omega}} + a = 0$$

$$\Rightarrow \quad (a - b\omega^2) + c\omega \cdot \overbrace{\sin\sigma\omega}^{=1} + ic\omega\,\underbrace{\cos(\sigma\omega)}_{=0}$$

volme: $\omega = \dfrac{\pi}{2\sigma} = \dfrac{\pi}{2c\tau}$

zbývá: $a - b\cdot\left(\dfrac{\pi}{2c\tau}\right)^2 + \dfrac{\pi}{2\tau} = 0$

$\Rightarrow$ $\exists!$ řešení $c = \dots$

$\Rightarrow$ $\exists$ cest. vlny :
(wavetrain)