3. Cestující vlny v rci reakce difuze

... dosud: $N' = f(N)$ ... $N = N(t)$ velikost populace v čase t

obecněji: $n = n(t,x)$ ... hustota pop. v (t,x)

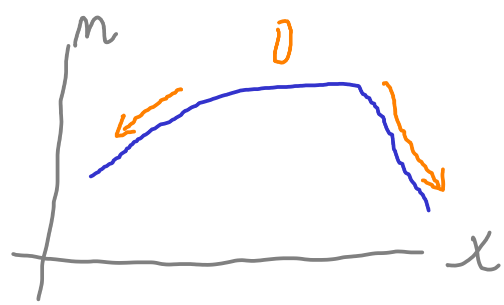
tj. $\mathcal{N}(I) = \int_I n(t,x)\,dx$ ... populace v oblasti I $(x \in I \subset \mathbb{R})$

r-d: odvození:

$$\frac{d}{dt}\mathcal{N}(I) = \underbrace{\mathcal{R}(I)}_{\text{reakce v } I} + \underbrace{\mathcal{D}(I)}_{\text{difuze z/do } I}$$

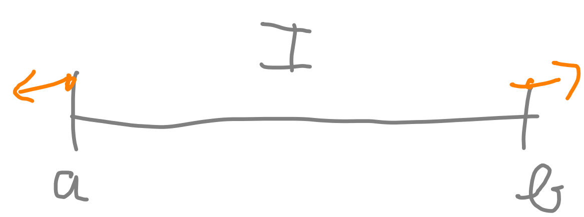
$$\mathcal{R}(I) = \int_I f(n(t,x))\,dx$$

?? $\mathcal{D}(I)$ ...



v bodě (vůči x) ... $-k\partial_x n$

tj. $\mathcal{D}(I) = k\partial_x n(t,b) - k\partial_x n(t,a)$



$$\frac{d}{dt}\int_I m\,dx = \int_I f(m)\,dx + k\underbrace{\left(\partial_x m|_b - \partial_x m_a\right)}_{\int_I \partial_{xx} m\,dx}$$

$$\int_I \left\{\partial_t m - f(m) - k\,\partial_{xx} m\right\} dx = 0, \quad \forall I$$

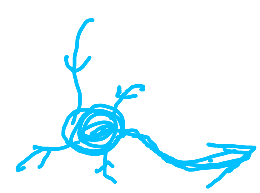
$\Rightarrow \{\dots\} = 0$, tj.

$$\boxed{\partial_t m = f(m) + k\,\partial_{xx} m}$$

Pozn.: ∂_{xx} … jiné odvození: neuronová síť

$u = u(t,x)$ … „el. potenciál" v bodě (t,x)

$$\partial_t u = -\alpha u + \int_{-\infty}^{\infty} k(y)\,u(x-y)\,dy$$



aproximace: $u(x-y) \doteq u(x) - y\,\partial_x u(x) + \frac{1}{2}y^2\,\partial_{xx} u(x)$

$$\Rightarrow \int_{-\infty}^{\infty} \dots = \int_{-\infty}^{\infty} k(y)\cdot\left[u(x) - y\,\partial_x u(x) + \frac{1}{2}y^2\,\partial_{xx} u(x)\right]$$

$$\partial_t u = -\alpha u + \int_{-\infty}^{\infty} k(y) \cdot \left[u(x) - y\partial_x u(x) + \frac{1}{2} y^2 \partial_{xx} u(x) \right] dy$$

$= 0$, $k(\cdot)$ sudé

$$a\, u(x) + b\, \partial_{xx} u(x)$$

kde
$$a = \int_{-\infty}^{\infty} k(y)\,dy$$

$$b = \int_{-\infty}^{\infty} \frac{1}{2} y^2 k(y)\,dy$$

$$\Rightarrow \quad \partial_t u = (a-\alpha) u + b\, \partial_{xx} u$$

R-D obecně: $\partial_t u = F(u, \partial_{xx} u)$

$u = u(t,x)$, $x \in \mathbb{R}$

cíl: $\exists$ cestující vlny

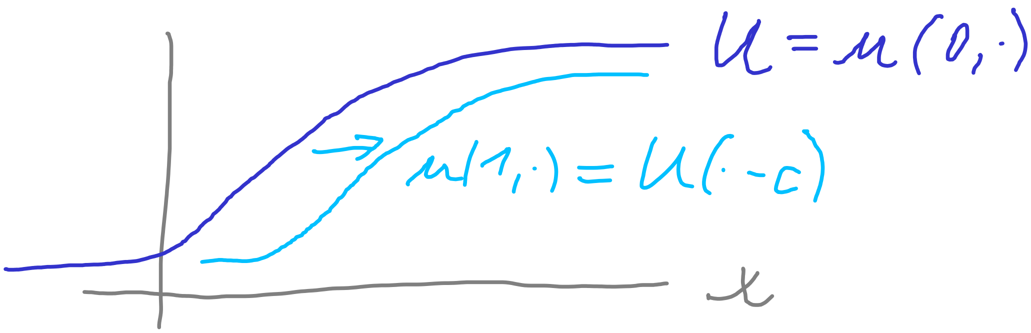
(speciální řešení)

Def. Řešením rce R-D ve tvaru cestující vlny rozumíme

$$u = u(t,x) = U(x-ct) \quad (*)$$

kde $U(s): \mathbb{R} \to \mathbb{R}$ je profil vlny

$c \neq 0$ ... rychlost vlny



Př. 1) vedení tepla: $\partial_t u = \partial_{xx} u$

dosaď: $*)$ LS: $\partial_t U(x-ct) = -c\,U'(x-ct)$

PS: $\partial_{xx} U(x-ct) = U''(\underbrace{x-ct}_{s})$

$$\Rightarrow \quad U''(s) + c\,U'(s) = 0$$

$\lambda^2 + c\lambda = 0 \quad \Rightarrow \quad$ F.S. $\{1, e^{-cs}\}$

$\lambda = 0, -c$

:(pouze konstantní nebo neomezené profily ...

2) vlnová rovnice: $\partial_{tt} u = \kappa \partial_{xx} u$

dosaď *)

$$(-c)^2 U'' = \kappa U''$$

$$(c^2 - \kappa) U'' = 0$$

⇒ lze vzít $U(\cdot)$ libovolné, pokud $c = \pm\sqrt{\kappa}$

3) Fisherova rce: $\partial_t u = \varkappa \partial_{xx} u + r\left(1 - \frac{u}{K}\right) u$

(logistický model + difuze)

buď $u(t,x) = U(x - ct)$ ⇒

$$-c U' = \varkappa U'' + r\left(1 - \frac{U}{K}\right) U$$

BÚNO: $\varkappa = r = K = 1$, polož: $V = U'$

$$\Rightarrow \quad U' = V$$

$$V' = U(U-1) - cV$$

?? ∃ netriv. leč omezené řešení...