cvičení 19.5.2020

Plán: 1) d.ú.3

(poznámky)

2) hra HDBR

(úvod)

3) různé

Pozn: RD + mutace, $x(0) \in \Delta$

$\Rightarrow \exists!$ řešení $\forall t \geq 0$

(navíc: $x(t) \in \operatorname{int} \Delta$, $\forall t > 0$)

Dk.: $x_i(t) = 0 \Rightarrow x_i'(t) > 0$

Př.: d.ú.3: V.D + opak + mutace

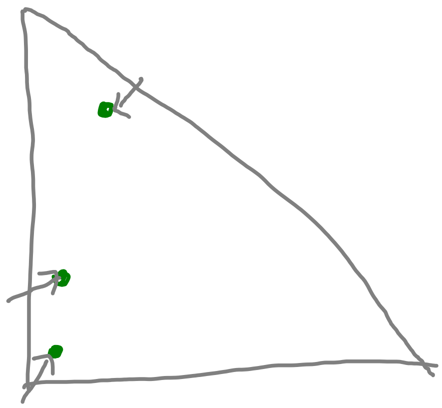
$$x' = \frac{1-\theta}{4} x(1-x-y)(5y-4x-4) + \theta\left(\frac{1}{3}-x\right)$$

$$y' = \frac{1-\theta}{4} y(1-x-y)(5y-4x-1) + \theta\left(\frac{1}{3}-y\right)$$

$\theta > 0$ malé

numerika:

$\theta = 0.05$



? ověření (analytické)

Pozn: teorie perturbací

$$\mathcal{F}(X,\theta)=0 \quad , \ \theta \ldots \text{malé}$$

$$\mathcal{F}(X,0)=0 \ \ldots \text{umíme vyřešit} \ (X_0)$$

tušíme: $\theta \neq 0$ (malé) $\Rightarrow \exists\, \tilde{X} = \tilde{X}(\theta) = X_0 + \theta X_1 + \ldots$

perturbace
- regulární: $\boxed{\text{VoIF}}$ ($D_X\mathcal{F}$ reg.)
- singulární

$\Uparrow$??

Aplikace: $\underbrace{F(X,\theta)=0}_{\text{P.S. rce}}$, $X = \begin{pmatrix} x \\ y \end{pmatrix}$

$$F: \mathbb{R}^2 \times \mathbb{R} \to \mathbb{R}^2$$

1) bod $P=\binom{0}{0}$: $\nabla_x F(0,0)=\begin{pmatrix}-1 & 0\\ 0 & -1/4\end{pmatrix}$

VoIF $\Rightarrow$ $\exists\ \tilde{X}=\tilde{X}(\theta)$ ... hladké

t. ž. $\boxed{F(\tilde{X}(\theta),\theta)=0,}$ $\forall\theta$ malé (*)

$\tilde{X}(0)=0$

?? poloha (i)

stabilita (ii)

ad (i) : rutinní aplikace VoIF

$$\frac{d}{d\theta}(*) \Rightarrow \nabla_x F(\tilde{X}(\theta),\theta)\tilde{X}'(\theta)+\frac{\partial F}{\partial\theta}(\tilde{X}(\theta),\theta)=0$$

$\theta=0$

$$\underbrace{\nabla_x F(0,0)}_{A_0}\,\tilde{X}'(0)+\underbrace{\frac{\partial F}{\partial\theta}(0,0)}_{\begin{pmatrix}1/3\\1/3\end{pmatrix}}=0$$

$$\boxed{A_0=\begin{pmatrix}-1 & 0\\ 0 & -1/4\end{pmatrix}}$$

$\Rightarrow$ $\tilde{X}'(\theta)=\begin{pmatrix}1/3\\4/3\end{pmatrix}$; tj.

$$\tilde{X}(\theta)=\binom{0}{0}+\frac{\theta}{3}\binom{1}{4}+o(\theta),\quad \theta\to 0$$

ad (iii) : **Pozn.** $A \in \mathbb{R}^{2\times 2}$ $\sigma(A) = \{\lambda_1, \lambda_2\}$

L.A. $\Rightarrow$

$$\lambda_1 + \lambda_2 = \operatorname{tr} A \quad \text{(stopa = součet diag.)}$$
$$\lambda_1 \lambda_2 = \det A$$

Tvrdíme: 1) $\operatorname{Re} \lambda_{1,2} < 0 \iff \operatorname{tr} A < 0$, $\det A > 0$

(stab. uzel)

2) $\left.\begin{array}{l} \lambda_1 < 0 < \lambda_2 \\ \text{nebo} \\ \lambda_2 < 0 < \lambda_1 \end{array}\right\} \iff \det A < 0$

(sedlo)

Dk.: rozborem případů (dle znamének $\lambda_{1,2}$ pozor na $\mathbb{C}$)

Zpět k příkladu:

$$A_\theta = (\nabla_x F)(\tilde{x}(\theta), \tilde{\theta}) = A_0 + \underbrace{\theta \cdot A_1 + \cdots}_{\text{netřeba!}}$$

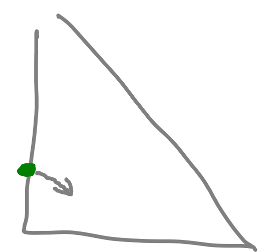
matice linearizace
(v pert. počátku)

víme: $A_0 = \begin{pmatrix} -1 & 0 \\ 0 & -1/4 \end{pmatrix}$ ... $\mathrm{tr}\, A_0 = -\frac{5}{4} < 0$

$\det A_0 = \frac{1}{4} > 0$

$\boxed{\theta \text{ malé}} \Rightarrow$ $\mathrm{tr}\, A_\theta < 0$, $\det A_\theta > 0$ $\Rightarrow \tilde{X}(\theta)$ asympt. stab.

bod: $\begin{pmatrix} 0 \\ 1/5 \end{pmatrix}$ ... $\tilde{X}'(0) = \begin{pmatrix} \frac{5}{9} \\ -\frac{2}{9} \end{pmatrix}$



ad segment $[x, 1-x]$; $x \in [0,1]$

TRIK: $F(\underline{\underline{x, y}}, \theta) = 0$

$$\underbrace{1 - x}_{X_1} - \underbrace{y}_{X_3} = \underbrace{\theta u}_{X_2}, \quad \text{tj. } \boxed{y = 1 - x - \theta u}$$

$$\frac{1-\theta}{4} x(1-x-y)(5y-4x-4) + \theta\left(\frac{1}{3} - x\right) = 0$$

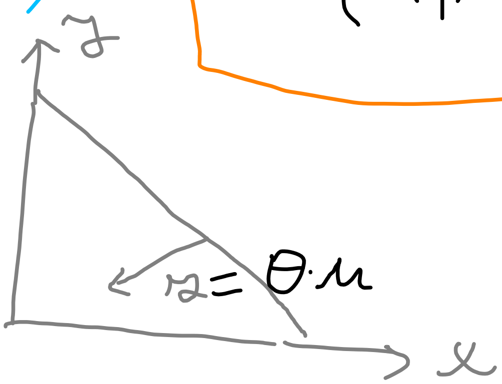
$$\frac{1-\theta}{4} y(1-x-y)(5y-4x-1) + \theta\left(\frac{1}{3} - y\right) = 0$$

$$\frac{1-\theta}{4}\, x \cdot \cancel{\theta} u \cdot \left(5(1-x-\theta u)-4x-4\right) + \cancel{\theta}\left(\frac{1}{3}-x\right) = 0$$

$$\frac{1-\theta}{4}\underbrace{(1-x-\theta u)}_{\cancel{\theta} u}\cdot\left(5(1-x-\theta u)-4x-1\right) + \cancel{\theta}\left(\frac{1}{3}-(1-x-\theta u)\right) = 0$$

$$\Rightarrow \quad \boxed{G(x,u,\theta) = 0} \quad (**)$$

... regulární perturbace



Pozn: chci $\theta u > 0$, $\theta > 0$ (malé) (tj. $u > 0$)

$(**)$, $\theta = 0$:

$$\boxed{\begin{aligned} \frac{xu}{4}(1-9x) &= x-\frac{1}{3} \\ \frac{1-x}{4}\, u\,(4-9x) &= \frac{2}{3}-x \end{aligned}}$$

podělíme:

$$\frac{x(1-9x)}{(1-x)(4-9x)} = \frac{x-\frac{1}{3}}{\frac{2}{3}-x}$$

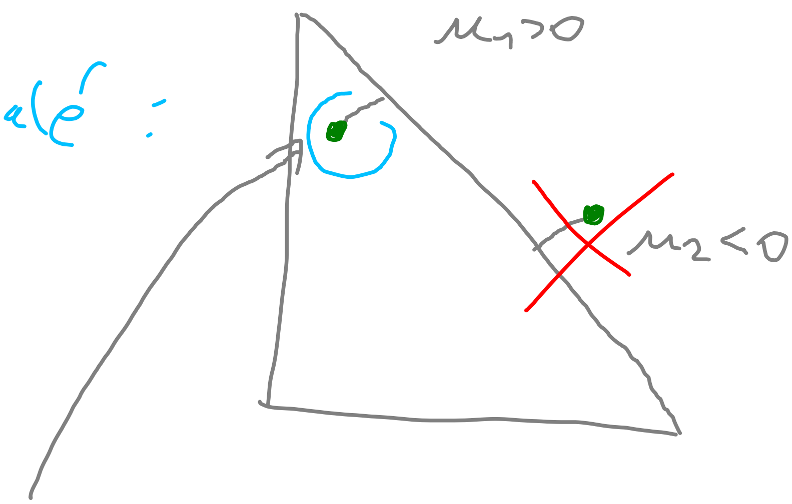
$\Rightarrow$ kvadr. rce: $x_{1,2} = \dfrac{23 \pm \sqrt{97}}{54}$

$$= \begin{cases} 0.24 \\ 0.61 \end{cases}$$

1. rce:

$$u_{1,2} = \begin{cases} 0.39 \\ -0.12 \end{cases}$$

$\theta > 0$ malé:



? stabilita $\tilde{x}(\theta)$

tj. $\nabla_x F(\tilde{x}(\theta), \theta) =: A_\theta$

$\theta > 0$ malé: $\det A_\theta > 0$

$\operatorname{tr} A_\theta < 0$ $\Rightarrow$ asympt. stabilita