A. Vyšetřete stejnoměrnou konvergenci následujících posloupností funkcí:

1. $f_n(x) = \dfrac{n^2x^2}{1+n^2x^2}$, 2. $f_n(x) = \dfrac{x^n}{1+x^n}$, $x > -1$,

3. $f_n(x) = x^n - x^{n+1}$, $x > -1$, 4. $f_n(x) = \sqrt[n]{1+x^n}$, $x \geq 0$,

5. $f_n(x) = \exp(-(x-n)^2)$, 6. $f_n(x) = x^n \exp(-nx)$, $x \geq 0$,

7. $f_n(x) = x^\alpha \exp(-nx)$, $x \geq 0$, $*$8. $f_n(x) = \left(1+\dfrac{x}{n}\right)^n$,

9. $f_n(x) = \dfrac{nx}{1+n+x}$, $x \geq -1$, 10. $f_n(x) = \sin(\pi x^n)$, $|x| \leq 1$,

11. $f_n(x) = \dfrac{\arctan(nx)}{nx}$, $x > 0$, 12. $f_n(x) = \dfrac{x}{n}\ln\left(\dfrac{x}{n}\right)$, $x > 0$,

13. $f_n(x) = n\left(\sin(x+\dfrac{1}{n}) - \sin x\right)$, $*$14. $f_n(x) = \sqrt[2n]{x^n + |\ln x|}$, $x > 0$,

15. $f_n(x) = n\ln\left(1+\dfrac{x}{n}\right)$, $x > -1$, 16. $f_n(x) = \dfrac{1+x^{n+1}}{1+x^n}$, $x > -1$,

17. $f_n(x) = \sqrt[n]{x+n}$, $x > -1$, 18. $f_n(x) = \dfrac{\ln(nx)}{nx}$, $x > 0$.

B. Vyšetřete (absolutně) stejnoměrnou konvergenci řad $\sum_k f_k(x)$, kde

1. $f_k(x) = k^p z^k,\ z \in C,\ k \in Z,$ 2. $f_k(x) = \sin\left(\frac{x}{2^k}\right),$

$*$3. $f_k(x) = \frac{kx^{k-1}}{1+x^k} \sin\left(\frac{x}{k}\right),\ x \in (0,1),$ 4. $f_k(x) = \frac{(-1)^k}{k^{1+\frac{1}{2}}} \frac{x^k}{1+x^k},$

5. $f_k(x) = \frac{(-1)^k}{kx^2+1},$ 6. $f_k(x) = \left(x^2+k^2\right)^{-1/2} \cos\frac{2k\pi}{3},$

$*$7. $f_k(x) = (-1)^k \frac{\sqrt{k}}{k+100} \frac{\cosh(kx)+\sinh(kx)}{\cosh(kx)},$ 8. $f_k(x) = (-1)^k(1-x)x^k,$

9. $f_k(x) = \sin^2 x \cos^k x,$ 10. $f_k(x) = \sin\left(x^{ke}\exp(-kx)\right),\ x \geq 0,$

11. $f_k(x) = \exp(-kx)\sin(kx^2),\ x \geq 0,$ 12. $f_k(x) = \frac{kx}{k^2+x^2} \operatorname{arctg}\left(\frac{x}{k}\right),$

$*$13. $f_k(x) = \operatorname{arccotg}\left(k^x + k^{1/x}\right),\ x \geq 0,$ 14. $f_k(x) = \frac{\sin x \sin(kx)}{\sqrt{x+k}},\ x \geq 0,$

15. $f_k(x) = \operatorname{arctg}\left(\frac{2x}{x^2+k^3}\right),$ 16. $f_k(x) = x^\alpha \exp(-kx),\ \alpha > 0,\ x \geq 0,$

17. $f_k(x) = \exp(-k^2 x),$ 18. $f_k(x) = \sin\left(\pi\sqrt{\alpha^2+k^2}\right) \sqrt[k]{\frac{x^2}{1+x^2}}.$