List of topics for the state doctoral examination (P4M3)
– differential equations and potential theory

Last updated: 2023/10/17 by D. Pražák

List A - general topics.

RA.1 **Qualitative theory of ODEs.**

stability, asymptotic stability, La Salle invariance principle, Lyapunov functions, principle of linearized (in)stability, Hartman-Grobman theorem, stable and unstable manifold

reference: Amann [1, Chapter IV, pp. 198–274]

examiners: D. Pražák, T. Bárta, O. Minakov

RA.2 **Classical potential theory.**

harmonic functions: the mean value property, Poisson's integral on a ball, Harnack's inequalities, convergence of harmonic functions, analyticity; subharmonic functions: equivalent conditions, approximation, relation to convexity and the mean value property; harmonic majorants, convergence of subharmonic functions

reference: Armitage & Gardiner [2, Chapter 1, pp. 1–30 and Chapter 3, pp. 59–84]

examiners: J. Spurný, P. Kaplický

RA.3 **Hyperbolic conservation laws.**

basic concepts from the theory of hyperbolic conservation laws, space of BV functions, Radon measures; nonlinear scalar conservation laws: existence and uniqueness of solutions; entropy, Rankine - Hugoniot conditions; energy methods for solving evolution equations

reference: Málek & al., Renardy & Rogers [11, pp. 1–10, 27–29, 33-63 and 281–287] and [14, pp. 377–384]

examiners: O. Kreml, M. Rokyta, E. Feiereisl

RA.4 **Optimal control theory.**

control problems: existence, uniqueness, continuity and differentiability of solutions; equivalent formulation using differential inclusions, closedness of the solution set; Kalman matrix - global linear and local nonlinear controllability conditions; existence of optimal control, first order necessary conditions: Pontryagin's maximum principle for Meyer's problem

reference: Bressan & Piccolli [3, Chapter 3-5 and Section 6.1-6.3, pp. 35–115]

examiners: D. Pražák, T. Bárta, O. Minakov

RA.5 **Sturm-Liouville theory for second order linear equations.**

linear homogeneous second order ODE - basic properties, Sturm comparison theorem, Sturm separation theorem; estimates of the number of zeros, non-oscillatory equations and principal solutions; asymptotic integration; disconjugate systems

reference: Hartman [10, Chapter XI, pp. 322–400]

examiners: D. Pražák, P. Kaplický, T. Bárta

RA.6 **Integral equations and eigenvalue problems.**

integral equation, kernel, homogeneous and adjoint problem; applications to Laplace equation, Neumann problem; compact and symmetric kernels, eigenvalues, Hilbert-Schmidt theorem

reference: DiBenedetto [6, Chapter IV, pp. 161–224]

examiners: D. Pražák, P. Kaplický, T. Bárta

RA.7 **Laplace transform.**

Stieltjes integral, domain of convergence of Laplace integral, absolute and uniform convergence, inverse Laplace transform, behavior of Laplace transform on vertical lines

reference: Widder [16, Chapter I-II, pp. 3–99]

examiners: T. Bárta, D. Pražák

RA.8 **Regularity of elliptic equations.**

Harnack inequality, Schauder estimates, nonlinear variational problem, Hilbert's 19th problem, De Giorgi - Nash theorem

reference: Fernández-Real & Ros-Oton [8, Chapter 1-3, pp. 1–97]

examiners: M. Bulíček, S. Schwarzacher, P. Kaplický

RB) Special topics.

RB.1 **Introduction to homogenization theory.**

homogenization of elliptic equations; method of multiple scales; Tartar's method of oscillating test functions; two-scale convergence method

reference: Cioranescu & Donato [4, Chapter 6-9, pp. 107–187]

examiners: P. Kaplický, Š. Nečasová

RB.2 **Basic theory of stochastic parabolic equations.**

linear equation with additive noise, linear equation with multiplicative noise, existence and uniqueness of solutions to nonlinear equations, martingale solutions *reference:* Da Prato & Zabczyk [5, Chapter 5-8, pp. 117–238]

examiners: J. Seidler

RB.3 **Existence theory for the Navier-Stokes-Fourier system.**

compressible viscous fluid with non-constant temperature; proof of the basic existence theorem - Galerkin approximation, artificial diffusion and pressure, a priori estimates, limit passages, compactness and compensated compactness; auxiliary tools: div-curl lemma, oscillation-defect measure

reference: Feireisl & Novotný [7, Chapter 3, pp. 44-126]

examiners: E. Feireisl, M. Pokorný, O. Kreml

RB.4 **Attractor: its structure and dimension estimates.**

semigroup of operators, dissipativity, global attractor - existence and its structure, upper and lower semicontinuity; example: reaction-diffusion equation, existence and regularity of the attractor; Hausdorff and box-counting dimension, differentiability of the semigroup, estimates of the attractor dimension

reference: Robinson [15, Chapter 10, 11, 13, pp. 259–306 and 325–352]

examiners: D. Pražák, P. Kaplický

RB.5 **Volterra integral equations.**

linear Volterra integral and integro-differential equations in R^n, local existence and uniqueness of solutions, asymptotic behavior of solutions (Paley-Wiener theorems), complete monotone kernels

reference: Gripenberg & al. [9, Chapters 2-3 and 5, pp. 35–89 and 140–168]

examiners: T. Bárta, D. Pražák, P. Kaplický

RB.6 **Regularity of Navier-Stokes equations.**

suitable weak solution and its existence; regular and singular point, estimates of the set of singular points; sufficient conditions of regularity: $L^\infty(0, T; L^3)$ and (lower) pressure estimates

reference: Pokorný [12, Chapter 3, pp. 33-63] a [13, s. 1–55]

examiners: M. Pokorný, P. Kaplický, J. Málek

References

[1] Herbert Amann. *Ordinary differential equations*, volume 13 of *de Gruyter Studies in Mathematics*. Walter de Gruyter & Co., Berlin, 1990. An introduction to nonlinear analysis, Translated from the German by Gerhard Metzen.

[2] David H. Armitage and Stephen J. Gardiner. *Classical potential theory*. Springer Monographs in Mathematics. Springer-Verlag London, Ltd., London, 2001.

[3] Alberto Bressan and Benedetto Piccoli. *Introduction to the mathematical theory of control*, volume 2 of *AIMS Series on Applied Mathematics*. American Institute of Mathematical Sciences (AIMS), Springfield, MO, 2007.

[4] Doina Cioranescu and Patrizia Donato. *An introduction to homogenization*, volume 17 of *Oxford Lecture Series in Mathematics and its Applications*. The Clarendon Press, Oxford University Press, New York, 1999.

[5] Giuseppe Da Prato and Jerzy Zabczyk. *Stochastic equations in infinite dimensions*, volume 44 of *Encyclopedia of Mathematics and its Applications*. Cambridge University Press, Cambridge, 1992.

[6] Emmanuele DiBenedetto. *Partial differential equations*. Birkhäuser Boston, Inc., Boston, MA, 1995.

[7] Eduard Feireisl and Antonín Novotný. *Singular limits in thermodynamics of viscous fluids*. Advances in Mathematical Fluid Mechanics. Birkhäuser Verlag, Basel, 2009.

[8] Xavier Fernández-Real and Xavier Ros-Oton. *Regularity theory for elliptic PDE*, volume 28 of *Zurich Lectures in Advanced Mathematics*. EMS Press, Berlin, 2022.

[9] G. Gripenberg, S.-O. Londen, and O. Staffans. *Volterra integral and functional equations*, volume 34 of *Encyclopedia of Mathematics and its Applications*. Cambridge University Press, Cambridge, 1990.

[10] Philip Hartman. *Ordinary differential equations*. S. M. Hartman, Baltimore, Md., 1973. Corrected reprint.

[11] J. Málek, J. Nečas, M. Rokyta, and M. Růžička. *Weak and measure-valued solutions to evolutionary PDEs*, volume 13 of *Applied Mathematics and Mathematical Computation*. Chapman & Hall, London, 1996.

[12] Milan Pokorný. Matematická teorie Navier-Stokesových rovnic. Skripta MFF UK.

[13] Milan Pokorný. Regularita Navier-Stokesových rovnic. Skripta MFF UK.

[14] Michael Renardy and Robert C. Rogers. *An introduction to partial differential equations*, volume 13 of *Texts in Applied Mathematics*. Springer-Verlag, New York, second edition, 2004.

[15] James C. Robinson. *Infinite-dimensional dynamical systems*. Cambridge Texts in Applied Mathematics. Cambridge University Press, Cambridge, 2001. An introduction to dissipative parabolic PDEs and the theory of global attractors.

[16] David Vernon Widder. *The Laplace Transform.* Princeton Mathematical Series, v. 6. Princeton University Press, Princeton, N. J., 1941.