NMFM401 – Mathematics of Non-Life Insurance 1

4. Frequency I – Models for the Number of Payments

A Exercises

1. The probability generating function (pgf) $P_N(z) = \mathbb{E}z^N$ has that name because

$$\mathbb{P}[N = k] = P_N^{(k)}(0)/k!.$$

That is, the probability of k observations can be obtained by evaluating the kth derivative of the pgf at zero. Demonstrate that this is true.

2. A portfolio of 10,000 risks produced the following numbers of claims.

No. of claims	No. of policies
0	9,048
1	905
2	45
3	2
4+	0

(a) Determine the MLE of λ for a Poisson model.

(b) Determine a 95% confidence interval for λ.

(c) Conduct the chi-squared goodness-of-fit test at $\alpha = 0.05$.

(d) Determine an approximate 95% confidence interval for λ based on the CLT.

3. Use the data from the previous exercise.

(a) Determine the MoM estimates of the parameters of the negative binomial model

$$\mathbb{P}[N = k] = \binom{k+r-1}{k}\beta^k(1+\beta)^{-k-r}, \quad k = 0, 1, \ldots; \; r > 0; \; \beta > 0.$$

(b) Conduct the chi-squared GoF test, using the MoM estimates.

(c) Determine the MLE of the parameters of the negative binomial distribution for this particular data set. [Hint: $\widehat{\mu} = \widehat{r}\widehat{\beta}$.]

(d) Conduct the likelihood ratio test to determine if a negative binomial model is more appropriate than a Poisson model.

4. Determine the MLE of β for the geometric distribution. Realize that the geometric distribution is the negative binomial with $r = 1$. In addition, determine the variance of the MLE.

B Homework

1. For the moment generating function (mgf) $M_N(z) = \mathbb{E}\exp\{zN\}$ demonstrate that $P_N(z) = M_N(\log z)$. Use the fact that $\mathbb{E}N^k = M_N^{(k)}(0)$ to show that $P'(1) = \mathbb{E}N$ and $P''(1) = \mathbb{E}[N(N-1)]$.

2. An automobile insurance policy provides benefits for accidents caused by both underinsured and uninsured motorists. Data on 1,000 policies revealed the following information.

No. of claims	Underinsured	Uninsured
0	901	947
1	92	50
2	5	2
3	1	1
4	1	0
5+	0	0

(a) Determine the MLE of λ for a Poisson model for each of the variables N_1 = number of underinsured claims and N_2 = number of uninsured claims.

(b) Conduct the chi-squared GoF test at $\alpha = 0.05$ for each model in the previous part.

(c) Assume that N_1 and N_2 are independent. Determine a model for $N = N_1 + N_2$.

3. An alternative method of obtaining a model for N in the previous exercise would be to record the total number of underinsured and uninsured claims for each of the 1,000 policies. Suppose this was done and results were as follows.

No. of claims	No. of policies
0	861
1	121
2	13
3	3
4	1
5	0
6	1
7+	0

(a) Determine the MLE of λ for a Poisson model.

(b) The answer to the previous part matched to the answer to part (c) of the previous exercise. Demonstrate that this must always be so.

(c) Conduct the chi-squared GoF test at $\alpha = 0.05$.

(d) Is this reason to believe (from these data) that N_1 and N_2 are independent? Does your answer agree with your intuition concerning this coverage?

4. Use the data from the previous exercise.

(a) Determine the MoM estimates of the parameter of the negative binomial model.

(b) Conduct the chi-squared GoF test, using the MoM estimates.

(c) Determine the MLE of the parameters of the negative binomial distribution.

(d) Conduct the likelihood ratio test to determine if a negative binomial model is more appropriate than a Poisson model.

(e) Based on these results, is there any reason to change your answer to part (d) of the previous exercise?

(f) Using the MLE, perform one iteration of the scoring method to verify that the likelihood has been maximized and to obtain an estimate of the asymptotic covariance matrix of $\widehat{\beta}$ and $\widehat{r}$.

(g) Use the covariance matrix obtained in the previous part to construct 95% confidence intervals for each parameter. Is there reason to believe that the two intervals are independent?

5. Consider a geometric model for each of the data sets in Exercises Ex 2 and Hw 3. For each data set do the following.

(a) Determine the MLE of β.

(b) Estimate the variance of $\widehat{\beta}$ and use it to construct an appropriate 95% confidence interval for β.

(c) Conduct a likelihood ratio test to determine if a negative binomial model is preferable to a geometric model.

(d) Of the Poisson, geometric, and negative binomial models, which is the best?

(Version: October 21, 2013)

Source

[1] Klugman, Stuart A., Panjer, Harry H., and Willmont, Gordon E. *Loss Models: From Data to Decisions.* John Wiley & Sons, Inc., New York, NY, 4th Edition, 2012.