- Find all the real solutions of the equations
  (1) $4^{x-1} + 4^{2-x} = 5$
  $x \in \{1; 2\}$. Introducing a new variable $y = 4^{x-1}$ leads to the quadratic equation $y^2 - 5y + 4 = 0$. Its solution $y = 4$ yields $x = 2$, and $y = 1$ gives $x = 1$.
  (2) $\log_3^2 x + \log_3 9^3 = \log_3 x^5$
  $x \in \{9; 27\}$. Put $y = \log_3 x$. Then $y \in \{2; 3\}$.
  (3) $\sin x - \sin(\pi + x) = 2\sin^2 x$
  $x \in \{k\pi;\ k \in \mathbb{Z}\} \cup \{\pi/2 + 2k\pi;\ k \in \mathbb{Z}\}$. Since $\sin(\pi + x) = -\sin x$, we can solve an equation in the variable $y = \sin x$. Do not forget to distinguish $\sin x = 0$ and $\sin x \neq 0$.
- Find all $x \in \mathbb{R}$ such that
  (1) $\frac{x-1}{x-4} > \frac{x-2}{x-3}$
  $x \in (5/2, 3) \cup (4, +\infty)$. The inequation is equivalent to $\frac{2x-5}{(x-3)(x-4)} > 0$. Now discuss negativity/positivity of the numerator and denominator, $x \neq 3; 4$.
  (2) $\frac{x^2-x-4}{x+1} \geq 0$
  $x \in \left\langle \frac{1-\sqrt{17}}{2}, -1 \right) \cup \left\langle \frac{1+\sqrt{17}}{2}, +\infty \right)$. Notice that $x \neq -1$.
  (3) $\big|4 - |x-3|\big| < 2$
  $x \in (-3, 1) \cup (5, 9)$.
  (4) $|x+1| + |x+3| < 4$
  $x \in (-4, 0)$. Discuss the positivity/negativity of $x + 1$ and $x + 3$.
  (5) $\log_2(x^2 + x + 6) > 0$
  $x \in \mathbb{R}$. The inequation is equivalent to $x^2 + x + 6 > 1$.
  (6) $x^2 + 1 - |x+2| > 0$
  $x \in \left(-\infty, \frac{1-\sqrt{5}}{2}\right) \cup \left(\frac{1+\sqrt{5}}{2}, +\infty\right)$. As an alternative approach, you may consider two functions $x^2 + 1$ and $|x+2|$, study the position of the graphs and compute their intersections.
  (7) $\cos^2 x + \frac{3}{2}\cos x - 1 < 0$
  $x \in \bigcup_{k\in\mathbb{Z}} \left(\frac{\pi}{3} + 2k\pi, \frac{5}{3}\pi + 2k\pi\right)$. Solve the respective quadratic equation in the variable $y = \cos x$. Of course, $y \in \langle -1, 1\rangle$. $y < \frac{1}{2}$ gives the result.
- Having given parameter $c \in \mathbb{R}$, determine all $x \in \mathbb{R}$ satisfying
  (1) $cx^2 + x + 1 > 0$
  $x > -1$ for $c = 0$. For $c > 0$: $x \in \left(-\infty, \frac{-1-\sqrt{1-4c}}{2c}\right) \cup \left(\frac{-1+\sqrt{1-4c}}{2c}, +\infty\right)$ for $c \in (0, 1/4\rangle$, $x \in \mathbb{R}$ for $c > 1/4$. $x \in \left(\frac{-1+\sqrt{1-4c}}{2c}, \frac{-1-\sqrt{1-4c}}{2c},\right)$ for $c < 0$.
  (2) $ce^x \in (-1, 0\rangle$
  No such x for $c > 0$, $x \in \mathbb{R}$ for $c = 0$, $x \in (\log(-1/c), +\infty)$ for $c < 0$.
  (3) $\log|x| + c \in (-\pi/2, \pi/2)$
  $x \in \left(-e^{\frac{\pi}{2}-c}, -e^{-\frac{\pi}{2}-c}\right) \cup \left(e^{-\frac{\pi}{2}-c}, e^{\frac{\pi}{2}-c}\right)$.
  (4) $|\cos x| - c > 0$
  $x \in \mathbb{R}$ for $c < 0$. $x \in \bigcup_{k\in\mathbb{Z}} \big(k\pi - \arccos c, k\pi + \arccos c\big)$ for $c \in \langle 0, 1)$. No such x for $c \geq 1$. Sketching the graph of cos is recommended.

(5) $e^{\sin x} - c \in (0, +\infty)$
$x \in \mathbb{R}$ for $c < 1/e$. $x \in \bigcup_{k\in\mathbb{Z}} \left(\arcsin \log c + 2k\pi, \pi - \arcsin \log c + 2k\pi\right)$ for $c \in \langle 1/e, e)$. No such x for $c \geq e$. Sketching the graph is recommended.

- Sketch the graphs of the functions

(1) $\left|\frac{3x+3}{2x-4}\right|$
$\frac{3x+3}{2x-4} = \frac{3}{2} + \frac{9}{2} \cdot \frac{1}{x-2}$ – the graph is a hyperbole with center $[2, 3/2]$, intersecting axes in $[0, -3/4]$ and $[-1, 0]$. To get the graph of $\left|\frac{3x+3}{2x-4}\right|$, consider the positive part of the previous graph with mirror image on the axis x of the negative part.

(2) $|\tan(-\pi x)|$
$|\tan(-\pi x)| = |-\tan(\pi x)| = |\tan(\pi x)|$. The graph of $\tan(\pi x)$ is the graph of tangent with primitive period 1 ($x \neq 1/2 + k$, $k \in \mathbb{Z}$). To get the graph of $|\tan(-\pi x)|$, consider the positive part of the previous graph with mirror image of the negative part.

(3) $|\sin(2-x) - 1|$
$|\sin(2-x) - 1| = |-\sin(x-2) - 1| = |\sin(x-2) + 1| = \sin(x-2) + 1$. Shift the graph of sine from $[0, 0]$ to $[2, 0]$ and then to $[2, 1]$.

(4) $\big|\log|x-1|\big|$
The graph of $\log|x|$ consists of two logaritmic curves (the graph of log and its reflection on the axis y). To get the graph of $\log|x-1|$, shift the previous one from $[0, 0]$ to $[1, 0]$. Then reflect its negative parts on the axis x.

- Analytic geometry:

(1) Let the line p be determined by the points $[0, 1]$ and $[1, 0]$. Compute the distance between p and the point $[\frac{1}{2}, 1]$.
$\frac{\sqrt{2}}{4}$. Sketching the situation is recommended.

(2) Express the intersection of the planes $x + y + z = 1$ and $x - z + 1 = 0$ in $\mathbb{R}^3$ as a line by vector equation (i.e. in the form $\vec{a} + t\vec{v}$, where $\vec{a}, \vec{v} \in \mathbb{R}^3$ and $t \in \mathbb{R}$ is a parameter).
There are many different expressions, e.g. $(0, 0, 1) + t(1, -2, 1)$, $t \in \mathbb{R}$. It suffices to find two different points, A, B of the intersection and then form $A + t.(B - A)$.

(3) Describe the curve given by the equality

$$4x^2 + y^2 - 16x - 8y + 16 = 0$$

and draw an appropriate picture.
$4x^2 + y^2 - 16x - 8y + 16 = 0$ is equivalent to $(4x^2 - 16x + 16) + (y^2 - 8x + 16) = 16$, i.e. $\left(\frac{x-2}{2}\right)^2 + \left(\frac{y-4}{4}\right)^2 = 1$ – the equation of ellipse with center $[2, 4]$ and major radius 4 (vertical), minor radius 2 (horizontal).