SKILLS AND KNOWLEDGE ONE SHOULD HAVE BEFORE STUDYING HIGHER MATHEMATICS

• To simplify and manipulate correctly with algebraic expressions. Use of the binomial theorem and similar rules.
• To handle trigonometric functions, exponential functions and logarithms. To know the geometric notion of trigonometric functions and their basic tabular values. It is not necessary to know all the angle sums by heart. One should know they exist and have certain structure, and estimate the situations when they can be helpful.
• To work with absolute value.
• To be able to solve equations and inequations: linear, quadratic, rational, with absolute value, simple equations and inequations involving elemetary functions. Each of them with/without parameter.

Sample problems:
• Find all the real solutions of the equations

$$4^{x-1} + 4^{2-x} = 5,$$
$$\log_3^2 x + \log_3 9^3 = \log_3 x^5,$$
$$\sin x - \sin(\pi + x) = 2\sin^2 x.$$

• Find all $x \in \mathbb{R}$ such that

$$\frac{x-1}{x-4} > \frac{x-2}{x-3},$$
$$\frac{x^2 - x - 4}{x+1} \geq 0,$$
$$\big|4 - |x-3|\big| < 2,$$
$$|x+1| + |x+3| < 4,$$
$$\log_2(x^2 + x + 6) > 0,$$
$$x^2 + 1 - |x+2| > 0,$$
$$\cos^2 x + \frac{3}{2}\cos x - 1 < 0.$$

• Having given parameter $c \in \mathbb{R}$, determine all $x \in \mathbb{R}$ satisfying

$$cx^2 + x + 1 > 0,$$
$$ce^x \in (-1, 0\rangle,$$
$$\log|x| + c \in (-\pi/2, \pi/2),$$
$$|\cos x| - c > 0,$$
$$e^{\sin x} - c \in (0, +\infty).$$

• To be able to solve a system of two linear equations in two variables.
• To draw graphs of linear fractional functions, simple polynomials, n-th root, trigonometric functions, exponential functions and logarithms. Also under 'shift' of the variable, its scalar multiple and with absolute value.

Exercises: Sketch the graphs of the functions

$$\left|\frac{3x+3}{2x-4}\right|, \quad |\tan(-\pi x)|, \quad |\sin(2-x)-1|, \quad \big|\log|x-1|\big|.$$

• Basic analytic (coordinate) geometry: dealing with vectors, different forms expressing straight lines and planes, vectors involved in the expressions, relative position of lines and planes, distance between a point and a line (a plane, respectively). Equations of circle, ellipse, parabola, hyperbola.

Sample problems:
• Let the line p be determined by the points $[0,1]$ and $[1,0]$. Compute the distance between p and the point $[\frac{1}{2},1]$.
• Express the intersection of the planes $x+y+z=1$ and $x-z+1=0$ in $\mathbb{R}^3$ as a line by vector equation (i.e. in the form $\vec{a}+t\vec{v}$, where $\vec{a},\vec{v}\in\mathbb{R}^3$ and $t\in\mathbb{R}$ is a parameter).
• Describe the curve given by the equality

$$4x^2+y^2-16x-8y+16=0$$

and draw an appropriate picture.

• Knowing the notion of complex number, its real and imaginary part, modulus (absolute value), complex conjugate, polar form of a complex number and its geometrical meaning (in complex plane), Moivre's formula.