MATHEMATICS I Exam, January 29, 2013

1. Compute the limit of the sequence or prove it does not exist:

$$\lim_{n \to \infty} \frac{n^2 \cdot \left(\log(n^2 + 1) - \log(n^2 + 2) \right) \cdot \operatorname{tg}\left(n^{-\frac{1}{3}}\right)}{\sqrt[3]{n^2 + 4n + 5} - \sqrt[3]{n^2 + 6n + 7}}.$$

13 points

2. Compute the limit of the function or prove it does not exist:

$$\lim_{x \to 0+} \left(x \cdot \cos x\right)^{\arcsin\left(\arcsin x\right)}.$$

12 points

3. Find domain of the given function, determine the set of all points in which it is continuous (continuous from the left/from the right). Compute its derivative (oneside derivatives) at each point where it exists.

$$f(x) = \begin{cases} \operatorname{arctg}\left(\frac{1}{x}\right) & \text{for } x \leq 1 \ \& \ x \neq 0, \\ -\frac{\pi}{2} & \text{for } x = 0. \end{cases}$$

15 points

4. Investigate properties of the function and draw its graph:

$$f(x) = e^{-2x^2 + 2x + \log 2}.$$

17 points

Write detailed arguments for each nontrivial step and verify assumptions of theorems you use.