MATHEMATICS I Exam, January 23, 2013

1. Compute the limit of the sequence or prove it does not exist:

$$\lim_{n\to\infty}\left(\frac{n+2}{n+1}\right)^{n+3}\cdot\frac{1}{n^4}\cdot\frac{\sqrt[n]{5^n+9^n}}{\sqrt[3]{n^6+7}-\sqrt[3]{n^6+8}}.$$

12 points

2. Compute the limit of the function or prove it does not exist:

$$\lim_{x\to 0+}\left(1+\sin(2\operatorname{tg} x)\right)^{\frac{x^2}{1-\cos(3x\sqrt{x})}}.$$

13 points

3. Find domain of the given function, determine the set of all points in which it is continuous (continuous from the left/from the right). Compute its derivative (oneside derivatives) at each point where it exists.

$$f(x)=\begin{cases}\operatorname{sgn} x+|\sin x| & \text{for } x\neq 0,\\ 1 \quad \text{for } x=0.\end{cases}$$

15 points

4. Investigate properties of the function and draw its graph:

$$f(x)=\frac{1}{x}+2\operatorname{arctg} x$$

17 points

Write detailed arguments for each nontrivial step and verify assumptions of theorems you use.