Cvičení 3 (19.10.2023)

La Salle.

1. (Dominik Bednář) In the domain $\Omega = (0,1)\times(0,1)$ find stationary points and study their stability
$$\begin{cases} x' = \frac{(2y-1)(1-x)x}{c+xy+(1-x)(1-y)}, \\ \\ y' = \frac{(1-2x)(1-y)y}{c+(1-x)y+x(1-y)}, \qquad c>0. \end{cases}$$
Hint. La Salle with $V(x,y) = -\left(\ln x + \ln(1-x) + \ln y + \ln(1-y)\right)$. $\dot V = \frac{-(2x-1)^2(2y-1)^2}{D_1\cdot D_2}$.

2. $\begin{cases} x' = y, \\ y' = -\alpha y x^2 - 2x, \quad \alpha > 0 \end{cases}$

   $[V = 2x^2 + y^2]$

3. Let $f, g \in C^1(\mathbb{R})$ are growing functions. Study stability of stationary points of the system $\begin{cases} x' = -y - f(x), \\ y' = g(x). \end{cases}$

   [Hint: $V = \int_0^x g(\tilde x)d\tilde x + \frac{1}{2}y^2$.]

4. Find ω-limit set:
$$\begin{cases} x' = y - x^7(x^4+2y^2-10), \\ y' = -x^3 - 3y^5(x^4+2y^2-10). \end{cases}$$
[Hint: $V = (x^4+2y^2-10)^2$.]

Poincaré-Bendixson, Poincaré-Dulac. Establish existence or non-existence of periodic orbits:

4. $\begin{cases} x' = 1 - xy, \\ y' = x, \end{cases}$

5. $\begin{cases} x' = x + xy^2, \\ y' = \frac{1}{2}(1-y^2), \end{cases}$

6. $\begin{cases} x' = \frac{x-2y}{1+x^2+y^2}, \\ y' = \frac{2x-\frac{1}{2}y}{1+x^2+y^2}. \end{cases}$

7. $\begin{cases} x' = y, \\ y' = -\alpha y - v'(x) \end{cases}$

   $[B = 1]$

8. $\begin{cases} x' = x - x^2 - xy, \\ y' = \frac{1}{2}y - \frac{1}{4}y^2 - \frac{3}{4}xy, \end{cases}$

   $[B = x^{-1}y^{-2}]$

9. $\begin{cases} x' = x(2-x-y), \\ y' = y(4x-x^2-3), \ x,y>0. \end{cases}$

   $[B = \frac{1}{xy}]$

10. $\begin{cases} x' = y, \\ y' = -x - y + x^2 + y^2. \end{cases}$

    $[B = e^{-2x}]$

11. van der Pol equation

    $x'' + (x^2-1)x' + x = 0.$

12. $\begin{cases} x' = -y + x(1-x^2-2y^2), \\ y' = x + y(1-2x^2-y^2). \end{cases}$

13. ??? (Jan Schwartz) $z' = \alpha(z + a\overline{z}^2 - ba^2 z|z|^2) + (-1 - e^{i\beta})z,\ z\in\mathbb{C},\ \alpha, a, b \in \mathbb{R}.$