Eulerovské substituce

$$\int R\left(x, \sqrt{x^2+2bx+c}\right) dx$$

VERZE 1

- $u = \sqrt{x^2+2bx+c} - x = \dfrac{2bx+c}{\sqrt{x^2+2bx+c}+x}$
- $u + x = \sqrt{x^2+2bx+c}$; $u^2 + 2ux + \cancel{x^2} = \cancel{x^2} + 2bx + c$; $\boxed{x = \dfrac{u^2-c}{-2(u-b)}}$
- $\boxed{dx} = \dfrac{2u(u-b) - (u^2-c)}{-2(u-b)^2} du = \boxed{\dfrac{u^2-2bu+c}{-2(u-b)^2} du}$
- $\boxed{\sqrt{x^2+2bx+c}} = u + x = u - \dfrac{u^2-c}{2(u-b)} = \dfrac{2u^2-2bu-u^2+c}{2(u-b)} = \boxed{\dfrac{u^2-2bu+c}{2(u-b)}}$

VERZE 2

- $v = \sqrt{x^2+2bx+c} + x = \dfrac{2bx+c}{\sqrt{x^2+2bx+c}-x}$
- $v - x = \sqrt{x^2+2bx+c}$; $v^2 - 2vx + \cancel{x^2} = \cancel{x^2} + 2bx + c$; $x = \dfrac{v^2-c}{2(v+b)}$
- $dx = \dfrac{2v(v+b) - (v^2-c)}{2(v+b)^2} dv = \dfrac{v^2+2bv+c}{2(v+b)^2} dv$
- $\sqrt{x^2+2bx+c} = v - x = v - \dfrac{v^2-c}{2(v+b)} = \dfrac{2v^2+2bv-v^2+c}{2(v+b)} = \dfrac{v^2+2bv+c}{2(v+b)}$