Konvergence Newtonova integrálu, týden 8, cvičení 14

Vyšetřete konvergenci následujících integrálů

1. $\displaystyle\int_0^1 \frac{dx}{e^x - \cos x}$

2. $\displaystyle\int_0^{\frac{\pi}{2}} \frac{dx}{\cos x\sqrt{\sin x}}$

3. $\displaystyle\int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{\cos x}\sqrt{\sin x}}$

4. $\displaystyle\int_0^{\frac{\pi}{2}} \tan x\, dx$

5. $\displaystyle\int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{\tan x}}$

6. $\displaystyle\int_0^1 x^{-10x}\, dx$

7. $\displaystyle\int_0^1 \frac{\log(1-x^2)}{x^2\sqrt{1-x^2}} dx$

8. $\displaystyle\int_0^\infty (\pi - 2\arctan(x))\, dx$

9. $\displaystyle\int_0^1 \frac{\arccos x\, dx}{\log^2(1/x)}$

10. $\displaystyle\int_1^2 \frac{\sqrt{x-1}\, dx}{\sqrt{x} - \sqrt[3]{x}}$

11. $\displaystyle\int_1^\infty \sqrt[4]{\exp\left(\frac{1}{x^2}\right) - \exp\left(\frac{-1}{x^2}\right)} \cdot \frac{\log(x+1)}{x+1}\, dx$

12. $\displaystyle\int_0^1 \log(\operatorname{arctg} x) \cdot \frac{\pi - 2\arcsin(x)}{(e^{1-x}-1)^2}\, dx$

13. $\displaystyle\int_0^5 \frac{\log(x^2 - 10x + 26)\, dx}{x\, e^{1/x}\, (5-x)^{5/2}}$

14. $\displaystyle\int_1^\infty \frac{\left(1 - \cos\frac{1}{x}\right)^{3/4}}{\sqrt{\sin\frac{1}{x}}} \operatorname{arctg}\left(3 + \frac{\log x}{x}\right) dx$

15. $\displaystyle\int_0^1 \frac{e^{2x^2} - e^{x^2}}{x^3\sqrt{\sin x}} \cdot \log(2 + \operatorname{arctg} x)\, dx$

Konvergence Newtonova integrálu, týden 8, cvičení 14

Vyšetřete konvergenci následujících integrálů

1. $\displaystyle\int_0^1 \frac{dx}{e^x - \cos x}$

2. $\displaystyle\int_0^{\frac{\pi}{2}} \frac{dx}{\cos x\sqrt{\sin x}}$

3. $\displaystyle\int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{\cos x}\sqrt{\sin x}}$

4. $\displaystyle\int_0^{\frac{\pi}{2}} \tan x\, dx$

5. $\displaystyle\int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{\tan x}}$

6. $\displaystyle\int_0^1 x^{-10x}\, dx$

7. $\displaystyle\int_0^1 \frac{\log(1-x^2)}{x^2\sqrt{1-x^2}} dx$

8. $\displaystyle\int_0^\infty (\pi - 2\arctan(x))\, dx$

9. $\displaystyle\int_0^1 \frac{\arccos x\, dx}{\log^2(1/x)}$

10. $\displaystyle\int_1^2 \frac{\sqrt{x-1}\, dx}{\sqrt{x} - \sqrt[3]{x}}$

11. $\displaystyle\int_1^\infty \sqrt[4]{\exp\left(\frac{1}{x^2}\right) - \exp\left(\frac{-1}{x^2}\right)} \cdot \frac{\log(x+1)}{x+1}\, dx$

12. $\displaystyle\int_0^1 \log(\operatorname{arctg} x) \cdot \frac{\pi - 2\arcsin(x)}{(e^{1-x}-1)^2}\, dx$

13. $\displaystyle\int_0^5 \frac{\log(x^2 - 10x + 26)\, dx}{x\, e^{1/x}\, (5-x)^{5/2}}$

14. $\displaystyle\int_1^\infty \frac{\left(1 - \cos\frac{1}{x}\right)^{3/4}}{\sqrt{\sin\frac{1}{x}}} \operatorname{arctg}\left(3 + \frac{\log x}{x}\right) dx$

15. $\displaystyle\int_0^1 \frac{e^{2x^2} - e^{x^2}}{x^3\sqrt{\sin x}} \cdot \log(2 + \operatorname{arctg} x)\, dx$

Výsledky.

1. D: v $x \to 0$ srovnáme s x^{-1}
2. D: v $x = 0$ K, v $x = \pi/2$ D
3. K
4. D: v $x = \pi/2$
5. K
6. K: není problém v $x = 0 : x^{10x} = e^{10x \log x} \to e^0 = 1$ pro $x \to 0$.
7. K: v 0 a 1 K
8. D: v ∞. Použijeme $\arctan x + \arctan \frac{1}{x} = \frac{\pi}{2} \mathrm{sgn} x, x \neq 0$.
9. D: D v 1, K v 0. Máme $\cos 0 = 1$, potom $\arccos 1 = 0$. Pro x blízko 1, definujeme $y = 1 - x$ a $\varphi = \arccos x$. Pak $\cos \varphi = x$, nebo $1 - \varphi^2/2 + \omega(\varphi)\varphi^2 = 1 - y$, pak $\varphi = \sqrt{2y}(1 + \omega_2(y))$.
10. K
11. K. Na nekonečnu srovnáme s $\dfrac{\log x}{\sqrt{x} \cdot x}$
12. D: K v 0, D v 1.
13. K: K v 0 díky $e^{-1/x}$, K v 5 díky $\log(1 + (x-5)^2) = (x-5)^2(1 + \omega(x))$, takže srovnáme s $(5-x)^{-1/2}$.
14. D: na nekonečnu srovnáme s $(\frac{1}{x^2})^{3/4}\sqrt{x} = x^{-1}$.
15. D: v 0 srovnáme s $\dfrac{x^2}{x^3\sqrt{x}} = x^{-3/2}$.