$$f(x)=\sqrt[3]{x^3+|b|^3}$$. $D_f=\mathbb{R}$ symmetrie-ne periodicita , průniky s osami xy; $f(0)=|b|$, $\lim\limits_{x\to+\infty} f(x)$

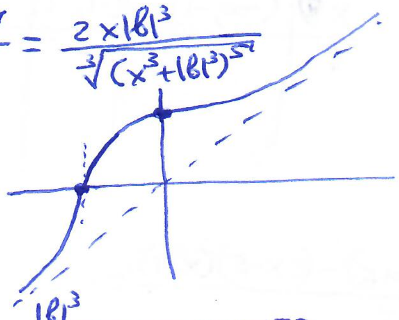
①

$$f'(x)=\frac{x^2}{\sqrt[3]{(x^3+|b|^3)^2}};\quad f'(0)=0,\ f\uparrow(-\infty,+\infty),\ f'_+(-b)=+\infty,\ f'_-(-b)=+\infty,\ f(-b)=0$$

$$f''(x)=\frac{2x}{\sqrt[3]{(x^3+|b|^3)^2}}-\frac{2x^2\cdot3x^2}{3\sqrt[3]{(x^3+|b|^3)^{5/3}}}=\frac{2x(x^3+|b|^3)-2x^4}{\sqrt[3]{(x^3+|b|^3)^5}}=\frac{2x|b|^3}{\sqrt[3]{(x^3+|b|^3)^5}}$$

x	$(-\infty,-b)$	$b-$	$-b+$	$(-b,0)$		$(0,+\infty)$
f''	+	$+\infty$	$-\infty$	−	0	+
f	∪			∩		∪

inflexion point



$$\lim_{x\to\pm\infty}\frac{f(x)}{x}=1,\quad \lim_{x\to+\infty}(f(x)-x)=\lim_{x\to+\infty}\left(\sqrt[3]{x^3+|b|^3}-x\right)=\lim_{x\to+\infty}\frac{|b|^3}{\sqrt[3]{(x^3+|b|^3)^2}+\sqrt[3]{(x^3+|b|^3)}\cdot x+x^2}=0$$

$y=x$ – asymptota pro $x\to+\infty$

$y=x$ – asymptota pro $x\to-\infty$

obor hodnot: $\mathbb{R}$

$f(x)=e^{-x^2}$ $D_f=\mathbb{R}$, $f(x)=f(-x)$

$f(0)=1$, $f\neq0$, $\lim\limits_{x\to\pm\infty} f(x)=0$

$f'(x)=-e^{-x^2}\cdot 2x$

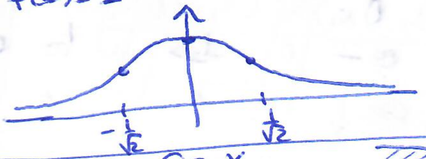
x	$(-\infty,0)$	0	$(0,+\infty)$
f'	+	0	−
f	↑		↓

$x=0$ – lok max, dokonce glob max

$f(0)=1$

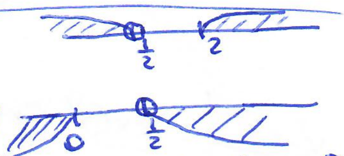
$f''(x)=e^{-x^2}(4x^2-2)$

x	$(0,\frac{1}{\sqrt2})$		$(\frac{1}{\sqrt2},+\infty)$
f''	−	0	+
f	∩		∪
	konkávní		konvexní



$f(x)=\arcsin\frac{1+x}{1-2x}$

D_f: $-1\le\frac{1+x}{1-2x}\le1$; $\frac{2x-1}{1-2x}\le\frac{1+x}{1-2x}\le\frac{1-2x}{1-2x}$; $\begin{cases}\frac{2-x}{1-2x}\ge0\\ \frac{3x}{1-2x}\le0;\end{cases}$

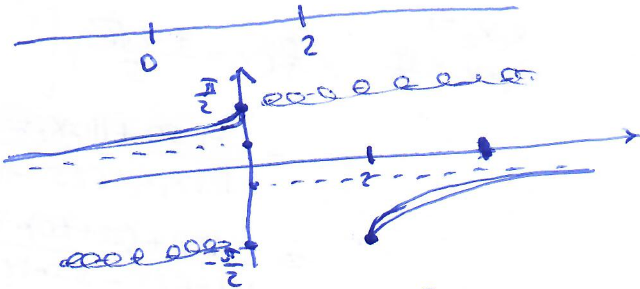


D_f: $x\in(-\infty,0]\cup[2,+\infty)$; $f(0)=\frac{\pi}{2}$, $f(2)=-\frac{\pi}{2}$; $\lim\limits_{x\to+\infty}f(x)=-\frac{\pi}{6}$, $\lim\limits_{x\to-\infty}f(x)=-\frac{\pi}{2}$

$f(D_f)\subset[-\frac{\pi}{2},\frac{\pi}{2}]$

$$f'(x)=\frac{1}{\sqrt{1-\left(\frac{1+x}{1-2x}\right)^2}}\cdot\frac{(1-2x)+2(1+x)}{(1-2x)^2}=\frac{3}{(1-2x)^2}\cdot\frac{|1-2x|}{\sqrt{(1-2x)^2-(1+x)^2}}=\frac{3}{|1-2x|\sqrt{-6x+3x^2}}=\frac{\sqrt3}{2\sqrt{x(x-2)}\,|x-\frac12|}$$

x	$(-\infty,0)$	$0-$		$2+$	$(2,+\infty)$
f'	+	$+\infty$		$+\infty$	+
f	↑				↑



$$f''(x)=\left(x^{-\frac12}(x-2)^{-\frac12}(x-\tfrac12)^{-1}\right)'=-\frac12x^{-\frac32}(x-2)^{-\frac12}(x-\tfrac12)^{-1}$$
$$-\frac12x^{-\frac12}(x-2)^{-\frac32}(x-\tfrac12)^{-1}$$
$$-x^{-\frac12}(x-2)^{-\frac12}(x-\tfrac12)^{-2}$$

$$=\frac{(x-2)(x-\frac12)+x(x-\frac12)+2x(x-2)}{-2x^{\frac32}(x-2)^{3/2}(x-\frac12)^2}=\frac{4x^2-7x+1}{-2x^{\frac32}(x-2)^{3/2}(x-\frac12)^2}$$

$(2x)^2-2\cdot(2x)\cdot\frac74+\frac{49}{16}-\frac{33}{16}$ $\left(2x-\frac74\right)^2=\frac{33}{16}$

$8x-7$: >0 $x>2$

<0 $x<0$

$$\left((-x)^{-\frac12}(2-x)^{-\frac12}(\tfrac12-x)^{-1}\right)'=\frac{-1\cdot(-1)}{2(-x)^{3/2}}(2-x)^{-\frac12}(\tfrac12-x)^{-1}-\frac12(-x)^{-\frac12}\cdot(2-x)^{-\frac32}\cdot(-1)(\tfrac12-x)^{-1}-(-x)^{-\frac12}(2-x)^{-\frac12}(\tfrac12-x)^{-2}\cdot(-1)=$$

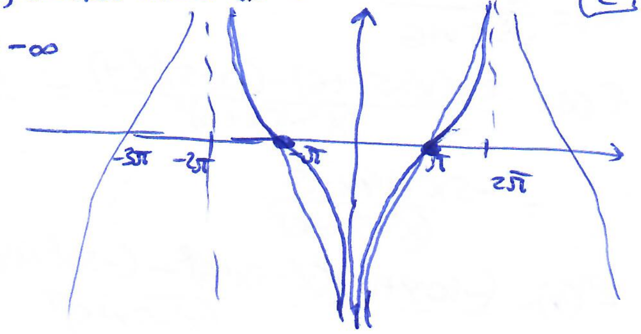
$$=\frac{(2-x)(\frac12-x)+(-x)(\frac12-x)+2(-x)(2-x)}{2(-x)^{\frac32}(2-x)^{3/2}(\frac12-x)^2}=\frac{(x^2-\frac52x+1)+(x^2-\frac12x)+2(x^2-2x)}{2(x(x-2))^{3/2}(\frac12-x)^2}=\frac{4x^2-7x+1}{2\sqrt{(x(x-2))^3}(x-\frac12)^2},\ x<0$$

$$f''(x)=\frac{(4x^2-7x+1)\cdot\operatorname{sgn}(x-\frac12)}{-2\sqrt{x^3(x-2)^3}\cdot(x-\frac12)^2},\ x\in(-\infty,0)\cup(2,+\infty)$$

x	$(-\infty,0)$	$0-$		$2+$	$(2,+\infty)$
f''	+	$+\infty$			−
f	∪				∩

$f(x)= \ln\left|\operatorname{tg}\frac{x}{4}\right|$ periodicita : 4π. Stačí $(-2\pi,2\pi)$, suda: stačí $(0,2\pi)$

$f(2\pi-)=+\infty$, $f(-2\pi+)=+\infty$, $f(0_+)=-\infty$



D_f: $\frac{x}{4}\neq \pi k_5+\frac{\pi}{2}$ $x\neq 4\pi k+2\pi$

$\operatorname{tg}\frac{x}{4}\neq 0$; $\frac{x}{4}\neq \pi k$ $x\neq 4\pi k$

$f(x)=0$: $\operatorname{tg}\frac{x}{4}=\pm 1$; $\frac{x}{4}=\pm\frac{\pi}{4}$; $x=\pi$

$$f'(x)=\frac{1}{4\operatorname{tg}\frac{x}{4}}\cdot\frac{1}{\cos^2\frac{x}{4}}=\frac{1}{4\sin\frac{x}{4}\cdot\cos\frac{x}{4}}=\frac{1}{2\sin\frac{x}{2}}$$

$x\in(0,2\pi)$, $\frac{x}{2}\in(0,\pi)$ $\sin\frac{x}{2}>0$

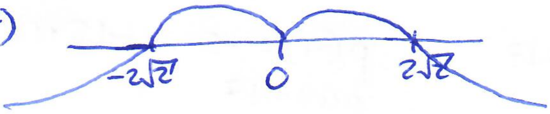
$f'(0_+)=+\infty$, $f'(2\pi_-)=+\infty$, $f'(\pi)=\frac{1}{2}$

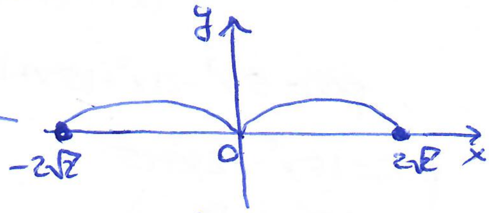
$$f''(x)=\frac{-\cos\frac{x}{2}}{4\sin^2\frac{x}{2}}$$

x	0	$(0,\pi)$	π	$(\pi,2\pi)$	2π
f''		$-$	0	$+$	
f		$\cap$		$\cup$	

$f(x)=\sqrt{8x^2-x^4}$ $x^2(8-x^2)$

suda





$D_f=[-2\sqrt{2},2\sqrt{2}]$

stačí $[0,2\sqrt{2}]$ $f(0)=0$, $f(2\sqrt{2})=0$

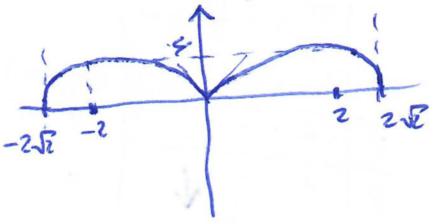
$f\geq 0$

$$f'(x)=\frac{8x-2x^3}{\sqrt{8x^2-x^4}}$$

$$=\frac{-2x(x^2-4)}{\sqrt{8x^2-x^4}}=\frac{-2(x^2-4)\operatorname{sgn}x}{\sqrt{8-x^2}}$$

x	0	$(0,2)$	2	$(2,2\sqrt{2})$	$2\sqrt{2}$
$f'(x)$	[illegible]	$+$	0	$-$	$-\infty$
f		$\uparrow$	4	$\downarrow$	

lok max



$$f''(x)=\frac{-4x}{\sqrt{8-x^2}}+\frac{-2(x^2-4)\cdot x}{(8-x^2)^{3/2}}=\frac{x\left[-4(8-x^2)-2(x^2-4)\right]}{(8-x^2)^{3/2}}=$$

$$=\frac{-2x\left(x^2-4+2(8-x^2)\right)}{(8-x^2)^{3/2}}=\frac{-2x(x^2-12)}{(8-x^2)^{3/2}}$$

4

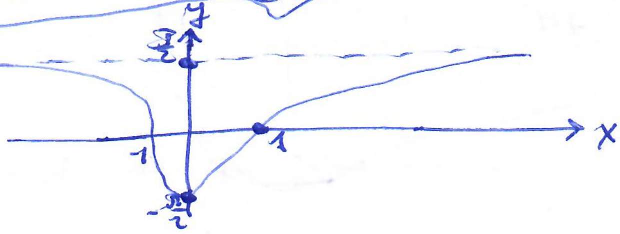
$$\left(\frac{(4-x^2)}{\sqrt{8-x^2}}\right)'=\frac{-2x}{\sqrt{8-x^2}}+(4-x^2)\cdot\frac{-1}{2}(8-x^2)^{-\frac{3}{2}}\cdot(-2x)$$

$$=\frac{-2x(8-x^2)+x(4-x^2)}{(8-x^2)^{3/2}}=\frac{-12x+x^3}{(8-x^2)^{3/2}}=\frac{x(x^2-12)}{(8-x^2)^{3/2}}$$

$f(x)=\arcsin\frac{x^2-1}{x^2+1}$ sudá

$f(0)=-\frac{\pi}{2}$, $-1\leq\frac{x^2-1}{x^2+1}\leq 1$

$\lim\limits_{x\to+\infty}f(x)=\frac{\pi}{2}$ $\frac{-x^2-1}{x^2+1}\leq\frac{x^2-1}{x^2+1}\leq\frac{x^2+1}{x^2+1}$; $\frac{x^2}{x^2+1}\geq 0$; $\frac{2}{x^2+1}\geq 0$

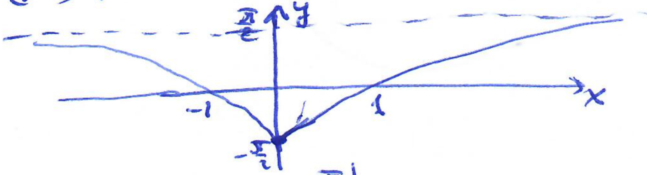


$f(x)=0 \Leftrightarrow \frac{x^2-1}{x^2+1}=0$; $x=\pm 1$; $f(1)=0$

$$f'(x)=\frac{1}{\sqrt{1-\left(\frac{x^2-1}{x^2+1}\right)^2}}\cdot\frac{2x(x^2+1)-2x(x^2-1)}{(x^2+1)^2}=\frac{4x}{(x^2+1)\sqrt{(x^2+1)^2-(x^2-1)^2}}=\frac{4x}{(x^2+1)\sqrt{4x^2}}=\frac{2\operatorname{sgn}x}{x^2+1},\ x\neq 0$$

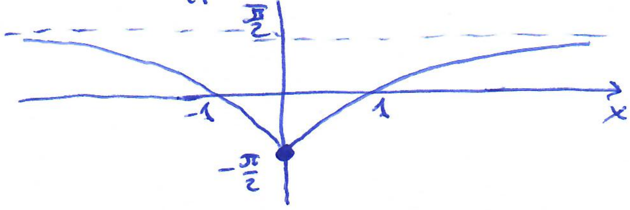
$f'_+(0)=1$, $f'_-(0)=-1$

x	$(0,+\infty)$
f'	$+$
f	$\uparrow$



$x>0$: $f''(x)=\frac{-4x\operatorname{sgn}x}{(x^2+1)^2}$

x	$(0,+\infty)$
f''	$-$
f	$\cap$



$f(x) = \frac{x^2-1}{x^2-5x+6} = \frac{(x-1)(x+1)}{(x-2)(x-3)}$ $D_f:\ x \neq 2,\ x \neq 3$

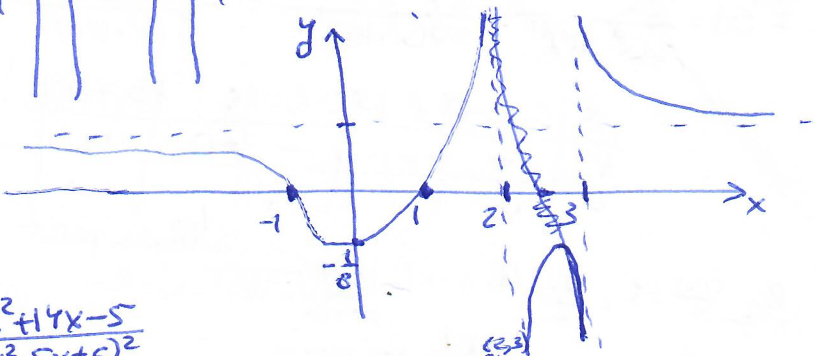
$\lim_{x\to+\infty} f(x) = 1$

$\lim_{x\to 3+} f(x) = +\infty$

x	$(-\infty,-1)$	$(-1, 1)$	$(1,2)$	$(2,3)$	$(3,+\infty)$
f	+	−	+	−	+

$f(x)=0 \Leftrightarrow x = \pm 1$

$f(0) = \frac{-1}{6}$



$f'(x) = \frac{2x(x^2-5x+6)-(2x-5)(x^2-1)}{(x^2-5x+6)^2} =$

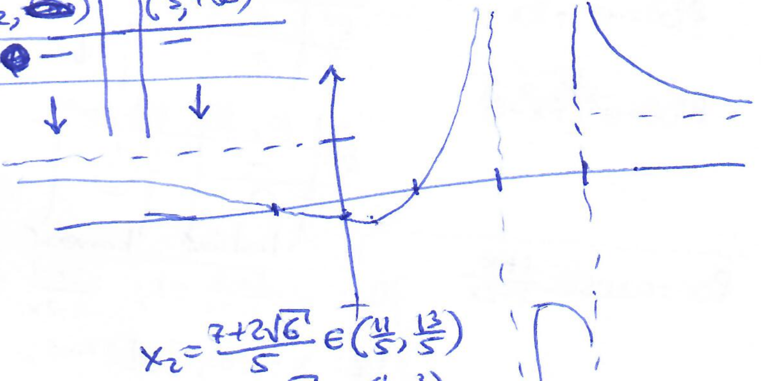
$= \frac{(2x^3-10x^2+12x)-(2x^3-5x^2-2x+5)}{(x^2-5x+6)^2} = \frac{-5x^2+14x-5}{(x^2-5x+6)^2}$

$= \frac{-5(x^2-\frac{14}{5}x+1)}{(x^2-5x+6)^2}$ $\left(x-\frac{7}{5}\right)^2 - \frac{49}{25} + \frac{25}{25} = 0;\ x - \frac{7}{5} = \pm\frac{2\sqrt{6}}{5};\ x_2 = \frac{7+2\sqrt{6}}{5} \in \left(\frac{11}{5}, \frac{13}{5}\right) \subset (2,3)$

$x_1 = \frac{7-2\sqrt{6}}{5} \in \left(\frac{1}{5}, \frac{3}{5}\right) \subset (0,1)$

x	$(-\infty, x_1)$	x_1	$(x_1;2)$	2_-	2_+	$(2, x_2)$	x_2	x_{2+}	$(x_2, 3)$		$(3,+\infty)$
$f'(x)$	−	0		$-\infty$	$+\infty$				−		−
f	↓		↑			↑			↓		↓

loc max



$\left(\frac{x^2-1}{x^2-5x+6}\right)' = \frac{2x(x^2-5x+6)-(x^2-1)(2x-5)}{(x^2-5x+6)^2}$

$= \frac{(2x^3-10x^2+12x)-(2x^3-5x^2-2x+5)}{(x^2-5x+6)^2} =$

$= \frac{-5x^2+14x-5}{(x^2-5x+6)^2} = \frac{-5(x^2-\frac{14}{5}x+1)}{(x^2-5x+6)^2} = \frac{-5(x-x_1)(x-x_2)}{(x-2)^2(x-3)^2}$

$x_2 = \frac{7+2\sqrt{6}}{5} \in \left(\frac{11}{5}, \frac{13}{5}\right)$

$x_1 = \frac{7-2\sqrt{6}}{5} \in \left(\frac{1}{5}, \frac{3}{5}\right)$

$x_1 \in \left(\frac{5}{13}, \frac{5}{11}\right)$

$x_1 x_2 = 1$

$x_1 + x_2 = \frac{14}{5}$

$f\left(x_1 = \frac{7-2\sqrt{6}}{5}\right) = \left[x_1^2 = \frac{14}{5}x_1 - 1\right] = \frac{\frac{14}{5}x_1 - 2}{\frac{14}{5}x_1 - 1 - 5x_1 + 6} = \frac{14x_1 - 10}{-11x_1 + 25} \cdot \frac{(-11x_2+25)}{(-11x_2+25)} =$

$= \frac{-250+110x_2+350x_1-154x_1x_2}{121x_1x_2+625-11\cdot25(x_1+x_2)} = \frac{-250-154+(110+350)\frac{7}{5}+\frac{2\sqrt{6}}{5}(110-350)}{121+625-275\cdot\frac{14}{5}} =$

$= \frac{-404+(22+70)\cdot7-4\sqrt{6}\cdot24}{121+625-55\cdot14} = \frac{490-404+154-96\sqrt{6}}{\underbrace{746-550-220}_{-24}} = \frac{240-96\sqrt{6}}{-24} =$

$= -10+4\sqrt{6} \in (-2,2)$

$\in(-2,0);\ 4\sqrt{6}\in(8,10)$

$\sqrt{6}\in(2,\frac{5}{2})$

$4\sqrt{6} \overset{<}{\vee} 10$

$2\sqrt{6} \vee 5$

$24 < 25$

$-10+4\sqrt{6} < -\frac{1}{6}$

$-60+24\sqrt{6} < -1$

$24\sqrt{6} < 59$

$24^2 \cdot 6 < (60-1)^2$

$(25-1)^2 \cdot 6$

$3600-120+1$

< 3481

$(625-50+1)\cdot6$

$(600-25+1)\cdot6$

$3600-150+6$

3456

$f\left(x_2 = \frac{7+2\sqrt{6}}{5}\right) = -10-4\sqrt{6} \in (-20,-18)$

$f''(x) = \frac{-10x+14}{(x^2-5x+6)^2} - \frac{(-5x^2+14x-5)\cdot2\cdot(2x-5)}{(x^2-5x+6)^3}$

$= \frac{1}{(x^2-5x+6)^3}\left[-(x^2-5x+6)(+10x-14)+(5x^2-14x+5)\cdot2(2x-5)\right]$

$= \frac{2}{(x^2-5x+6)^3}\left[-(x^2-5x+6)(5x-7)+(5x^2-14x+5)(2x-5)\right]$

$= \frac{2}{(x^2-5x+6)^3}\left[-(5x^3-32x^2+65x-42)+(10x^3-53x^2+80x-25)\right]$

$= \frac{2(5x^3-21x^2+15x+17)}{(x^2-5x+6)^3}$

$p(x) = 5x^3-21x^2+15x+17$

$p(2) = 40-84+30+17 = -44 \overset{+60}{} -13 \overset{+60}{} = -57\ 3$

$p(3) = 135-189+45+17 \overset{>}{} 0$

$$f(x)=\frac{x^2-1}{x^2-5x+6}$$

$$f'(x)=\frac{2x(x^2-5x+6)-(2x-5)(x^2-1)}{(x^2-5x+6)^2}=\frac{2x^3-10x^2+12x-(2x^3-5x^2-2x+5)}{(x^2-5x+6)^2}=$$

$$=\frac{-5x^2+14x-5}{(x^2-5x+6)^2}$$

$$f''(x)=\frac{(-10x+14)(x^2-5x+6)^2-(-5x^2+14x-5)\cdot 2(x^2-5x+6)\cdot(2x-5)}{(x^2-5x+6)^4}=$$

$$=\frac{(-10x+14)(x^2-5x+6)+(5x^2-14x+5)\cdot 2(2x-5)}{(x^2-5x+6)^3}=$$

$$=\frac{2}{(x-2)^3(x-3)^3}\left[(5x^2-14x+5)(2x-5)-(5x-7)(x^2-5x+6)\right]$$

$$10x^3-53x^2+80x-25-(5x^3-32x^2+65x-42)$$

$$=\frac{2(5x^3-21x^2+15x+17)}{(x-2)^3(x-3)^3}$$

$p(x)=5x^3-21x^2+15x+17$

$p(-1)=-5-21-15+17=-24$ $x_1\in(-1,0)$

$p(0)=17$

$p(1)=5-21+15+17$

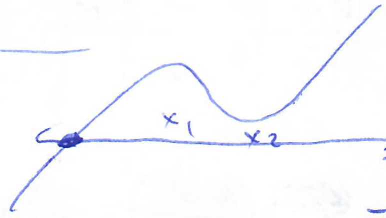
$p(2)=$

$p'(x)=15x^2-42x+15$

$=15(x^2-\frac{14}{5}x+1)$

x	$(-\infty,x_1)$	(x_1,x_2)	$(x_2,+\infty)$
p'	+	−	+
p	↑	↓	↑
		max	min

$x_2=\frac{7+2\sqrt{6}}{5}$



$p(x_2)$

$x_2^2=\frac{14}{5}x_2-1$

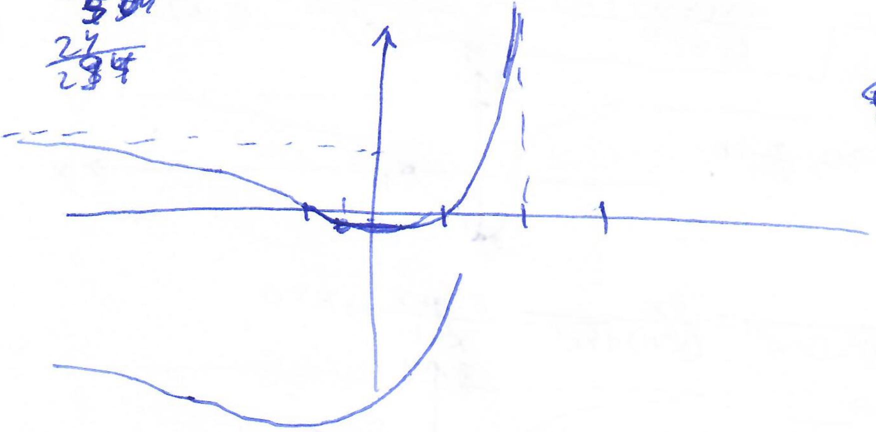
$p(x_2)=5x_2^3-21x_2^2+15x_2+17$

$=x_2(14x_2-5)-21(\frac{14}{5}x_2-1)+15x_2+17$

$=14x_2^2+x_2(15-5-\frac{6\cdot 49}{5})+38$

$\frac{50-294}{5}$

49
6
594
24
294



$14(\frac{14}{5}x_2-1)-\frac{244}{5}x_2+38$

$\frac{196}{5}x_2-\frac{244}{5}x_2-14+38$

$\frac{-48}{5}x_2+24=24(\frac{-2}{5}x_2+1)$

$=\frac{-48}{5}(x_2-\frac{5}{2})>0$

$x_2=\frac{7+2\sqrt{6}}{5}\overset{<}{\vee}\frac{5}{2}$

$14+4\sqrt{6}\vee 25$

$4\sqrt{6}\vee 11$

$16\cdot 6\vee 121$

$96<121$