Approximating the Navier–Stokes equations on $\mathbb{R}^3$ with large periodic domains

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I will consider solutions u_α of the three-dimensional Navier–Stokes equations on the periodic domains $Q_\alpha := (-\alpha, \alpha)^3$ as the domain size $\alpha \to \infty$, and compares them to solutions of the same equations on the whole space. For compactly-supported initial data $u_\alpha^0 \in H^1(Q_\alpha)$, an appropriate extension of u_α converges to a solution u of the equations on $\mathbb{R}^3$, strongly in $L^r(0, T; H^1(\mathbb{R}^3))$, $r \in [1, 4)$ (the result is in fact more general than this). The same also holds when u_α^0 is the velocity corresponding to a fixed, compactly-supported vorticity. Such convergence is sufficient to show that if an initial compactly-supported velocity $u_0 \in H^1(\mathbb{R}^3)$ or an initial compactly-supported vorticity $\omega_0 \in H^1(\mathbb{R}^3)$ gives rise to a smooth solution on $[0, T^*]$ for the equations posed on $\mathbb{R}^3$, a smooth solution will also exist on $[0, T^*]$ for the same initial data for the periodic problem posed on Q_α for α sufficiently large; this illustrates a 'transfer of regularity' from the whole space to the periodic case.