On nonlinear problems of parabolic type with implicit constitutive equations involving flux

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We study systems of nonlinear partial differential equations of parabolic type, in which the elliptic operator is replaced by the first order divergence operator acting on a flux function, which is related to the spatial gradient of the unknown through an additional implicit equation. This setting, broad enough in terms of applications, significantly expands the paradigm of nonlinear parabolic problems. Formulating four conditions concerning the form of the implicit equation, we first show that these conditions describe a maximal monotone p-coercive graph. We then establish the global-in-time and large-data existence of (weak) solution and its uniqueness. Towards this goal, we adopt and significantly generalize the Minty method of monotone mappings. A unified theory, containing several novel tools, is developed in a way to be tractable numerically.