Nonlinear Aggregation-Diffusion Equations: Gradient Flows, Free Energies and Phase Transitions

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The main goal of this talk is to discuss the state-of-the-art in understanding the phenomena of phase transitions for a range of nonlinear Fokker–Planck equations with linear and nonlinear diffusion. They appear as natural macroscopic PDE descriptions of the collective behavior of particles such as Cucker–Smale models for consensus, the Keller–Segel model for chemotaxis, and the Kuramoto model for synchronization. We will show the existence of phase transitions in a variety of these models using the natural free energy of the system and their interpretation as natural gradient flow structure with respect to the Wasserstein distance in probability measures. We will discuss both theoretical aspects as well as numerical schemes and simulations keeping those properties at the discrete level.