c_{-1} se nazývá <u>residuum fce f v bodě a</u>
ozn. $\boxed{\operatorname{res}_a f}$

Věta 15.14 <u>Residuová věta</u>

Buď $\underset{\text{otevř.}}{\Omega} \subset \overline{\Omega} \subset \underset{\text{otevř.}}{U} \subset \mathbb{C}$, Ω navíc omezená, a $\partial\Omega$
lze popsat jako konečný počet reg. hladkých orient. křivek
Nechť $A \subset \Omega$ konečná množina
a $f \in H(U \setminus A)$. Potom
$$\int_{\partial\Omega} f(z)\,dA = 2\pi i \sum_{a\in A} \operatorname{res}_a f$$

Tvrzení (jak určovat hodnoty residuí)

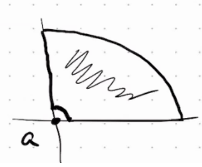
(1) Je-li $f \in H(B_\rho(a))$, pak $\operatorname{res}_a f = 0$

(2) Jsou-li $f, g \in H(B_\rho(a))$ a g má v a kořen násobnosti 1
pak $\operatorname{res}_a \frac{f}{g} = \frac{f(a)}{g'(a)}$

(3) Je-li $f \in H(B_\rho(a))$ a g má v a pól násobnosti 1
pak $\operatorname{res}_a fg = f(a)\operatorname{res}_a g$

(4) Má-li f v a pól násobnosti nejvýše m,
pak $\operatorname{res}_a f = \lim_{z\to a} \left[f(z)(z-a)^m \right]^{(m-1)} \frac{1}{(m-1)!}$

<u>Lemma</u> (o obcházení <u>jednonásobného</u> pólu)
Má-li f v a <u>pól násobnosti 1</u>, a je-li $\langle\varphi\rangle$ část kružnice
parametrizovaná $\varphi(t) = a + \varepsilon e^{it}$, $t \in \langle\alpha,\beta\rangle$ $\quad \langle\alpha,\beta\rangle \subsetneq \langle 0, 2\pi\rangle$
Pak $\int_\varphi f(z)\,dA \xrightarrow[\varepsilon\to 0]{} i(\beta-\alpha)\operatorname{res}_a f$



$0 \le a < b \le 2\pi$

<u>Lemma</u> (JORDANOVO)

- Buď $\langle\varphi\rangle : \langle a,b\rangle \to \mathbb{C}$ parametrizovaná $\quad \left[\begin{array}{l} \varphi : \langle a,b\rangle \to \mathbb{C} \\ \varphi(t) = Re^{it} \quad t\in\langle a,b\rangle \end{array}\right.$
- Nechť f je spojitá v $\mathbb{C} \setminus B_r(0)$ pro R dostatečně velké

Buď $M_R := \max_{z\in\langle\varphi\rangle} |f(z)|$

Pak

(a) Je-li $RM_R \to 0$ pro $R\to\infty$, pak $\int_{\langle\varphi\rangle} f(z)\,dA \to 0$ pro $R\to\infty$

(b) Je-li $M_R \to 0$ pro $R\to\infty$ a $\left\{\begin{array}{l} \langle a,b\rangle \subset \langle 0,\pi\rangle \text{ a } \gamma > 0 \\ \langle a,b\rangle \subset \langle -\pi, 0\rangle \text{ a } \gamma < 0 \end{array}\right\}$

pak $\int_{\langle\varphi\rangle} f(z) e^{i\gamma z}\,dA \to 0$ pro $R\to\infty$

(5) Integrály $\int_0^{2\pi} f(\cos t, \sin t)\,dt = \int_{|z|=1} F(z)\,dz$

$\cos t = \frac{e^{it}+e^{-it}}{2}$ $\quad z = e^{it}$

$\sin t = \frac{e^{it}-e^{-it}}{2i}$ $\quad dz = ie^{it}\,dt$

$\frac{dz}{z} = i\,dt$

(6) Integrály $I(a) = \int_0^\infty z^{a-1} f(z)\,dz$ Mellinova transformace

$a \in \mathbb{C} \setminus \mathbb{Z}$

- (7) $I = \int_0^1 x^{\alpha-1}(1-x)^{-\alpha}\,dx$

(8) Integrály typu $I = \int_0^\infty f(x) \ln x\,dx$

- f sudá
- f nemá pól v 0
- $\max |f(Re^{it})| R \ln R \xrightarrow[R\to\infty]{} 0$

$$\boxed{\text{Integrály typu } \int_0^{2\pi} R(\cos t, \sin t)\,dt \quad \Big|\; R(x,y) = \frac{P(x,y)}{Q(x,y)}}$$

$$\int_0^{2\pi} R\left(\frac{e^{it}+e^{-it}}{2}, \frac{e^{it}-e^{-it}}{2i}\right) dt$$

$$\int_\gamma f(z)\,dz = \int_0^{2\pi} f(e^{it})\,\underbrace{ie^{it}}_{(e^{it})'}\,dt = \underbrace{\int R(\cos t, \sin t)\,dt}$$

γ ← jednotková kružnice

+ označení $e^{it} = z$, $e^{-it} = \frac{1}{z}$

$$\int_0^{2\pi} R(\cos t, \sin t)\,dt = \int_0^{2\pi} R\left(\frac{e^{it}+e^{-it}}{2}, \frac{e^{it}-e^{-it}}{2i}\right) dt =$$

$$\boxed{= \int_{|z|=1} R\underbrace{\left(\frac{z+\frac{1}{z}}{2}, \frac{z-\frac{1}{z}}{2i}\right)}\frac{dz}{iz} = \int_0^{2\pi} R(\cos t, \sin t)\,dt}$$

$e^{it} = z$

$i e^{it}\,dt = dz$

$iz\,dt = dz$

$$\boxed{I=\int_0^{2\pi}\frac{dt}{a+\cos t}} \qquad \boxed{a>1}$$

$$\frac{1}{a+\cos t}$$

$$\cos t = \frac{e^{it}+e^{-it}}{2}$$

z

$$\cos t = \frac{z+\frac{1}{z}}{2}$$

$$\int_0^{2\pi}\frac{dt}{a+\cos t} = \int_{|z|=1} \frac{1}{a+\frac{z+\frac{1}{z}}{2}}\,\frac{1}{iz}\,dz$$

$$f(z) = \frac{1}{iz}\,\frac{1}{a+\frac{z+\frac{1}{z}}{2}} = \frac{1}{i}\,\frac{2}{z^2+2az+1} =$$

$$= \frac{2}{i}\,\frac{1}{z^2+2a+a^2-a^2+1} = \frac{2}{i}\,\frac{1}{(z+a)^2+1-a^2}$$

$$= \left|\, z+a = \pm\sqrt{\underbrace{a^2-1}_{>0}} \,\right| = \frac{2}{i}\,\frac{1}{\left((z+a)+\sqrt{a^2-1}\right)\left(z+a-\sqrt{a^2-1}\right)}$$

$$\int_{|z|=1} f(z)\,dt = 2\pi i \sum_{\text{uvnitř}} \operatorname{Res}_{z_i} f(z)$$

$z_1 = -a-\sqrt{\overbrace{a^2-1}^{>0}} < -a < -1$ — neleží uvnitř

$$z_2 = -a+\sqrt{a^2-1} \overset{?}{>} -1 \longleftrightarrow -a+\sqrt{a^2-1} > -1 \quad /\cdot(-1)$$

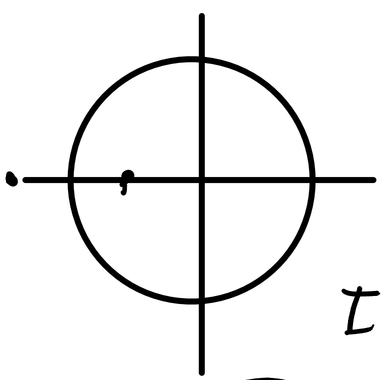
$$a-\sqrt{a^2-1} < 1$$

$$\underbrace{a-1}_{>0} < \sqrt{a^2-1}$$

$$a^2-2a+1 < a^2-1$$

$$2 < 2a$$

$$1 < a$$



$$I = 2\pi i\,\operatorname{Res}_{z_2} = 2\pi i\,\frac{2}{i}\,\frac{1}{2\sqrt{a^2-1}}$$

$$\frac{1}{i}\,\frac{2}{z^2+2az+1} = \qquad = \frac{2\pi}{\sqrt{a^2-1}}$$

$$2z+2a$$

$$\boxed{I = \int_0^{2\pi} \frac{\cos^2 t}{13+12\cos t}\,dt =} = \int_{|z|=1} \frac{1}{zi}\left(\frac{z+\frac{1}{z}}{2}\right)^2 \frac{1}{13+6\left(z+\frac{1}{z}\right)}\,dz =$$

$$= \frac{1}{4i}\int_{|z|} \overbrace{\left(\frac{z^2+1}{z}\right)^2 \frac{1}{13z+6z^2+6}}^{f(z)}\,dz$$

z^2 $\qquad z_1 = -\frac{2}{3}$

$z_0 = 0$ $\qquad \cancel{z_2 = -\frac{3}{2}} \leftarrow$

$$I = \int_{|z|=1} f(z)\,dz = 2\pi i\left(\mathrm{Res}_0 f(z) + \mathrm{Res}_{-\frac{2}{3}} f(z)\right)$$

$$\mathrm{Res}_{-\frac{2}{3}} \quad \frac{\left(\frac{4}{9}+1\right)^2}{\frac{4}{9}} \frac{1}{12\left(-\frac{2}{3}\right)+13} = \frac{13^2}{9\cdot 4}\,\frac{1}{5} \qquad \left|\ \frac{f(z_0)}{g'(z_0)} \leftarrow \frac{f(z)}{g(z)}\right.$$

$12z+13$

$$\mathrm{Res}_{z=0} f(z) = \lim_{z\to 0}\frac{d}{dz}\,\frac{1}{1!}\cdot\frac{(z^2+1)^2}{6z^2+13z+6}\cdot\cancel{\frac{z^2}{z^2}} =$$

$$\lim_{z\to 0}\frac{2\cdot 2z(z^2+1)(6z^2+13z+6)-(z^2+1)^2(12z+13)}{(6z^2+13z+6)^2} =$$

$$= -\frac{13}{36}$$

$$\sum_{\text{uvnitř}} \mathrm{Res}\, f(z) = \frac{13^2}{9\cdot 4}\cdot\frac{1}{5} - \frac{13}{36} = \frac{13}{36}\left(\frac{13-5}{5}\right) = \frac{2\cdot 13}{45}$$

$$I = \frac{2\pi i}{4i}\sum \mathrm{Res}\, f(z) = \frac{1}{4i}\,2\pi i\,\frac{2\cdot 13}{45} = \pi\,\frac{13}{45}$$

$$\boxed{I = \int_0^{2\pi} e^{\cos t}\cos(nt-\sin t)\,dt \qquad n\in \mathbb{N}}$$

$$e^{\cos t}\cos(nt-\sin t) = \mathrm{Re}\; e^{\cos t}\, e^{i(nt-\sin t)} =$$

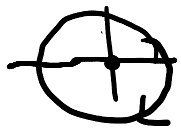
$$= \mathrm{Re}\; e^{\cos t - i\sin t}\, e^{int} = \mathrm{Re}\; e^{e^{-it}}\, e^{int}$$

$$I = \mathrm{Re}\int_0^{2\pi} \underbrace{e^{e^{-it}}}\, e^{int}\,dt = X$$

$$z = e^{-it}$$

$$dz = -i\, e^{it}\,dt = -i\, z\,dt$$

$t\in[0,2\pi]$



$$\oint f(z) = (-)2\pi i \sum \mathrm{Res}$$

$$X = \mathrm{Re}\; -\frac{1}{i}\int_{|z|=1} \underbrace{e^{z} z^{-n}\frac{1}{z}}_{f(z)}\,dz \qquad \underbrace{f(z) = e^{z} z^{-(n+1)}}_{n+1 \text{ násobný pól}}$$

$$\mathrm{Res}_0 f(z) = \underbrace{\frac{d^n}{dz^n}}_{\lim_{z\to 0}} \frac{1}{n!}\,\cancel{z^{-(n+1)}}\, e^{z}\, \cancel{z^{n+1}} = \frac{1}{n!}$$

$$I = \mathrm{Re}\left(-\frac{1}{i}\,(-2\pi i)\,\frac{1}{n!}\right) = \frac{2\pi}{n!}$$

Integrál typu

$$\boxed{I(\alpha)=\int_0^{\infty} x^{\alpha-1} f(x)\,dx}$$

$\alpha\in(0,2)$

$|f(z)|\le \frac{C}{|z|^2}$

$f(z)$ můžeme holomorfně rozšířit na $\mathbb{C}\setminus\{a_1,\dots,a_n\}$ $n\in\mathbb{N}$
a a_i ty neleží na kladné reálné ose ani v $z=0$.

musíme holomorfně rozšířit i $\underline{x^{\alpha-1}}$

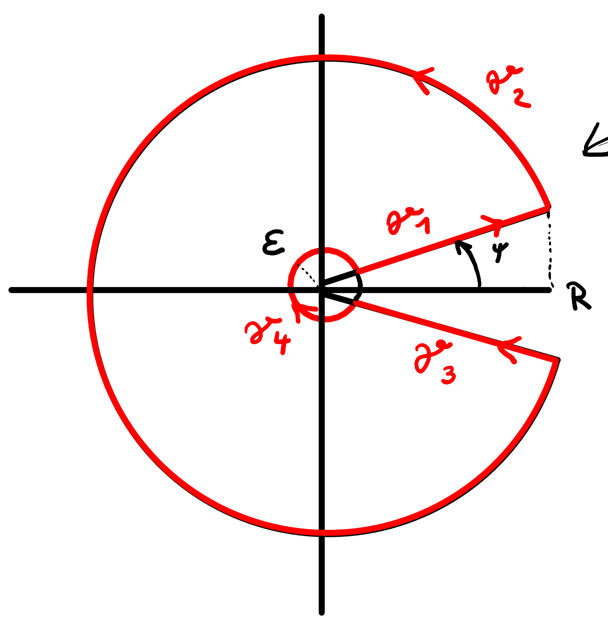
potřebujeme definovat z^{β} $\beta\in\mathbb{R}$

$$z^{\beta}:=e^{\beta\ln z}=e^{\beta(\ln|z|+i\arg z)}$$

volíme větev logaritmu tak, aby byl ln holomorfní na vnitřku zvolené křivky (volbu podřídíme volbě křivky)

$\boxed{(0,2\pi)}$

$$g(z)=z^{\alpha-1}f(z)$$



$\Gamma=\gamma_1+\gamma_2+\gamma_3+\gamma_4$

γ_1:

$\gamma_1(t)=te^{i\varphi}$ $t\in[\varepsilon,R]$

$\gamma_2(t)=Re^{it}$ $t\in[\varphi,2\pi-\varphi]$

$\gamma_3(t)=te^{i(2\pi-\varphi)}$

$\gamma_4(t)=\varepsilon e^{it}$ $t\in[2\pi-\varphi,\varphi]$

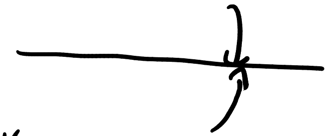
$$\int_{\Gamma(\varepsilon,R,\varphi)} z^{\alpha-1}f(z)\,dz=2\pi i\sum \operatorname{Res}_{a_i} z^{\alpha-1}f(z)$$

$\uparrow$ rezidua uvnitř křivky

γ_1, γ_3 a vztah k Γ

$z^{\alpha-1}$ $f(t)$

$[\varepsilon, R]$ $x>0$



γ_1: $\gamma_1(t) = t e^{i\psi}$ $t \in [\varepsilon, R]$

γ_3: $\gamma_3(t) = t e^{i(2\pi-\psi)}$ $t \in [R, \varepsilon]$

$$\int_{\gamma_1} z^{\alpha-1} f(z)\,dz = \int_\varepsilon^R \underbrace{(t e^{i\psi})^{\alpha-1}} f(t e^{i\psi}) \; e^{i\psi}\,dt$$

$$(t e^{i\psi})^{\alpha-1} = e^{(\alpha-1)(\ln|t e^{i\psi}| + i \arg t e^{i\psi})} =$$

$$= t^{\alpha-1} e^{i(\alpha-1)\psi} \xrightarrow[\psi\to 0^+]{} t^{\alpha-1}$$

$$\int_{\gamma_3} z^{\alpha-1} f(z)\,dz = -\int_\varepsilon^R \underbrace{(t e^{i(2\pi-\psi)})^{\alpha-1}} f(t e^{i\psi})\, e^{i(2\pi-\psi)}\,dt$$

$$(t e^{i(2\pi-\psi)})^{\alpha-1} = e^{(\alpha-1)(\ln|t e^{i(2\pi-\psi)}| + i \arg t e^{i(2\pi-\psi)})} =$$

$$= t^{\alpha-1} e^{i(\alpha-1)(2\pi-\psi)} \xrightarrow{\psi\to 0^+} t^{\alpha-1} e^{i(\alpha-1)2\pi}$$

$$= t^{\alpha-1} e^{2\pi i\alpha} \underbrace{e^{-2\pi i}}_{1} = t^{\alpha-1} \underbrace{e^{2\pi i\alpha}}$$

$$\int_{\gamma_1+\gamma_3} z^{\alpha-1} f(z)\,dz \xrightarrow{\psi\to 0^+} \int_\varepsilon^R t^{\alpha-1} f(t)(1 - e^{2\pi i\alpha})\,dt$$

$$\xrightarrow[R\to\infty]{\varepsilon\to 0^+} \Gamma(\alpha)(1 - e^{2\pi i\alpha})$$

$\alpha = n \in \mathbb{N}$ α nemůže být celé číslo

OBLOUKY $\gamma_2 + \gamma_4$

γ_2 (velký)

$$\left|\int_{\gamma_2} z^{\alpha-1} f(z)\, dz\right| \leq 2\pi R\, R^{\alpha-1} \frac{C}{R^L} = \tilde{C} R^{\alpha-L} \xrightarrow[\alpha-L<0]{R\to\infty} 0$$

$\boxed{\alpha < L}$

γ_4 malý

$$\left|\int_{\gamma_4} z^{\alpha-1} f(z)\, dz\right| \leq \underbrace{2\pi C\, \varepsilon^{\alpha-1}\cdot \varepsilon}_{\tilde{C}\quad \varepsilon^{\alpha}} \xrightarrow[\alpha>0]{\varepsilon\to 0^+} 0 \qquad \alpha > 0$$

$$\rightarrow \int_{\Gamma} z^{\alpha-1} f(z)\, dz = 2\pi i \sum_{j=1}^{k} \operatorname{Res}_{a_j}\left(f(z) z^{\alpha-1}\right)$$

$$I(\alpha)\left(1-e^{2\pi i\alpha}\right) = 2\pi i \sum_{j=1}^{k} \operatorname{Res}_{a_j}\left(f(z) z^{\alpha-1}\right)$$

$$\int_0^\infty x^{\alpha-1} f(x)\, dx = \boxed{I(\alpha) = \frac{2\pi i}{\left(1-e^{2\pi i\alpha}\right)} \sum \operatorname{Res}_{a_j}\left(f(z) z^{\alpha-1}\right)}$$

$$\boxed{I = \int_0^\infty \frac{dx}{(x+1)\sqrt{x}}}$$

$$I = \frac{2\pi i}{(1-e^{2\pi i \alpha})} \sum \operatorname{Res}_{z_j} z^{\alpha-1} f(z)$$

$$I = -\frac{\pi e^{-\pi i \alpha}}{\sin \pi\alpha} \sum \operatorname{Res}_{z_j} z^{\alpha-1} f(z)$$

$$\underbrace{\left(\frac{1}{x+1}\right)}_{f(x)} x^{-1/2} = \left(\frac{1}{x+1}\right) x^{\frac{1}{2}-1} \qquad \alpha \in (0,1)$$

$$\int_0^\infty x^{\alpha-1} f(x)\, dx \qquad \alpha = 1/2 \qquad f(x) = \frac{1}{x+1}$$

$$f(z) \leq \frac{C}{R} \qquad L = 1$$

$$\operatorname{Res}_{-1} f(z) = \operatorname{Res}_{-1} \underbrace{\frac{1}{z+1}}_{\frac{1}{f}} \underbrace{\frac{1}{(z)^{1/2}}}_{g} = \frac{1}{(-1)^{1/2}} = \frac{1}{e^{i\frac{\pi}{2}}}$$

$$I = \frac{2\pi i}{(1-e^{2\pi i \frac{1}{2}})} \frac{1}{e^{i\frac{\pi}{2}}} = \frac{2\pi i}{\underbrace{e^{\frac{\pi i}{2}}(e^{-\frac{i\pi}{2}} - e^{i\frac{\pi}{2}}) \; e^{i\frac{\pi}{2}}}} \frac{1}{} =$$

$$= \frac{2\pi i}{e^{\pi i}(e^{i\frac{\pi}{2}} - e^{-\frac{\pi i}{2}})} = -\frac{\pi}{\underbrace{e^{\pi i}}} \frac{1}{\sin\frac{\pi}{2}} = \frac{\pi}{\sin\frac{\pi}{2}}$$

Integrál typu $\boxed{I=\int_0^\infty x^{\alpha-1} f(x)\,dx}$ $\alpha\in(0,1)$

$|f(z)|\le \frac{C}{|z|}$

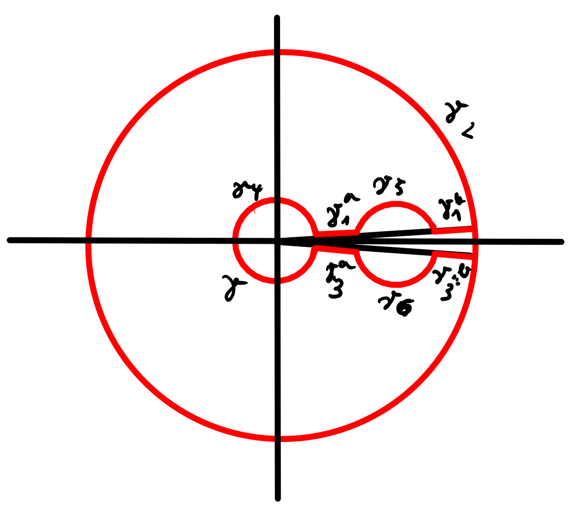
$f(z)$ můžeme holomorfně rozšířit na $\mathbb{C}\setminus\{a_1,\dots,a_n\}$ $n\in\mathbb{N}$
a a_i tj. neleží ~~na kladné reálné~~ ose ani v $z=0$.

zkusme rozšířit na jednoduché póly pro $x>0$ (umíme je obcházet!)

$\alpha\in(0,1)$

$|f(z)|\le \frac{C}{|z|}$

$f(z)$ má jednoduchý pól v bodě $\boxed{z=b}$
+ póly uvnitř oblasti (mimo kladnou reálnou osu)



$\gamma_1=\gamma_1^a+\gamma_1^b$

$\gamma_1:\ te^{i\varphi}$ $t\in[\varepsilon,b-\varepsilon]\cup[b+\varepsilon,R]$

$\gamma_3:\ te^{i(2\pi-\varphi)}$ $t\in[R,b+\varepsilon]\cup[b-\varepsilon,\varepsilon]$

$\to\gamma_5:\ b+\varepsilon e^{it}$ $t\in[\pi-\delta_b(\varphi),\tilde\delta_b(\varphi)]$

$\to\gamma_6:\ b+\varepsilon e^{it}$ $t\in[2\pi-\tilde\delta_b(\varphi),\pi-\delta_b(\varphi)]$

$\gamma_4:\ \varepsilon e^{it}$ $t\in[2\pi-\delta_0(\varphi),\delta_0(\varphi)]$

$\gamma_2:\ Re^{it}$ $t\in[\delta_R(\varphi),2\pi-\delta_R(\varphi)]$

γ_2 a γ_4 viz předchozí

γ_1 a γ_3:

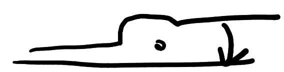
integrály ve smyslu v.p.
$$\int_\varepsilon^{b-\varepsilon} x^{\alpha-1} f(x)\,dx+\int_{b+\varepsilon}^{\infty} x^{\alpha-1} f(x)\,dx \xrightarrow{\varepsilon\to0^+} I$$

$\gamma_5: \int_{\gamma_5} f(z) z^{\alpha-1}\,dz = \int_{\pi-\delta(\varepsilon)}^{\delta(\varepsilon)} (b+\varepsilon e^{it})^{\alpha-1} (b+\varepsilon e^{it})'\, f(b+\varepsilon e^{it})\,dt$

1 nás. pól v b $\quad \lim_{z\to b} f(z)(z-b) = \operatorname{Res} f(z)$

$\lim_{\varepsilon\to 0^+} f(b+\varepsilon e^{it})\,\varepsilon e^{it}$

$(b+\varepsilon e^{it})^{\alpha-1} = e^{(\alpha-1)\left(\ln|b+\varepsilon e^{it}| + i\arg(b+\varepsilon e^{it})\right)}$

$\xrightarrow[\varepsilon\to 0^+]{} b^{\alpha-1} e^{(\alpha-1)\cdot i 0} =$ 

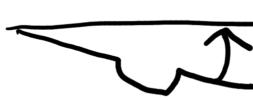
$= b^{\alpha-1}$

$\gamma_5 \quad \int z^{\alpha-1} f(z)\,dz \xrightarrow{\varepsilon\to 0^+} \int_{\pi}^{0} i\,\overbrace{(b+\varepsilon e^{it})^{\alpha-1}}^{b^{\alpha-1}} \times \underbrace{f(b+\varepsilon e^{it})\,\varepsilon e^{it}}_{\operatorname{Res}_b f(z)}\,dt$

$\xrightarrow{\varepsilon\to 0^+} -i\pi\, b^{\alpha-1} \operatorname{Res}_b f(z)$

$\gamma_6 \quad \int_{\gamma_6} f(z) z^{\alpha-1}\,dz = \int_{\gamma_6} \underbrace{f(b+\varepsilon e^{it})}\cdot \underbrace{(b+\varepsilon e^{it})^{\alpha-1}}\, i\underbrace{\varepsilon e^{it}}\,dt$

$(b+\varepsilon^{it})^{\alpha-1} = e^{(\alpha-1)\left(\ln|b+\varepsilon e^{it}| + i\arg(b+\varepsilon e^{it})\right)}$

 2π

$\xrightarrow{\varepsilon\to 0^+} b^{\alpha-1} \underbrace{e^{(\alpha-1)\, i 2\pi i}}$

$\int_{\gamma_6} f(z) z^{\alpha-1}\,dz \xrightarrow{\varepsilon\to 0^+} \int_{2\pi}^{\pi} i\,\underbrace{(b+\varepsilon e^{it})^{\alpha-1}}_{b^{\alpha-1}\cdot e^{\cdots}}\, \underbrace{f(b+\varepsilon e^{it})\,\varepsilon e^{it}}_{\operatorname{Res}_b f(z)}\,dt$

$$= -i\pi\, b^{\alpha-1}\, e^{(\alpha-1)2\pi i}\, \mathrm{Res}_b f(z)$$

$$= -i\pi\, b^{\alpha-1}\, e^{2\pi i \alpha}\, \mathrm{Res}_b f(z)$$

$$I(\alpha)\left(1-e^{2\pi i\alpha}\right) - i\pi \boxed{\left(1+e^{2\pi i\alpha}\right)} \mathrm{Res}_{b} f(z)$$

$$= 2\pi i \sum_{i=1}^{k} f(z)\, z^{\alpha-1}$$

$\longrightarrow$ póly jednonásobné na ose x $x>0$

$$\boxed{I(\alpha) = i\pi \frac{1+e^{2\pi i\alpha}}{1-e^{2\pi i\alpha}} \sum_{j=1}^{l} b_j^{\alpha-1} \mathrm{Res}_{b_j} f(z) + \frac{2\pi i}{1-e^{2\pi i\alpha}} \sum_{j=1}^{k} \mathrm{Res}_{a_j} z^{\alpha-1} f(z)}$$

$$\boxed{\int_0^\infty \frac{dx}{(x-1)\sqrt{x}}}$$

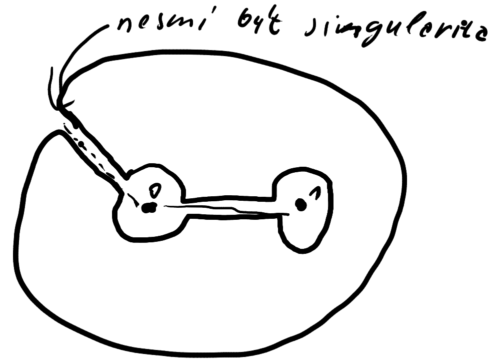
$$I = i\pi \, \frac{\overbrace{1 + e^{i\pi 2 \frac{1}{2}}}^{0}}{\underbrace{1 - e^{i\pi}}_{2}} \cdot 1^{1/2-1} \operatorname{Res}_1(1/z) = 0$$

Integrály typu

$$\boxed{\int_0^1 x^{\alpha-1}(1-x)^{-\alpha} g(x)\,dx}$$

$-1 < \alpha < 2$

možnost 1 integrační oblast typu činka



možnost 2 → REÁLNÁ substituce

$$\frac{x^{\alpha-1}}{(1-x)^{\alpha}} g(x) = \frac{x^{\alpha}}{(1-x)^{\alpha}} \frac{1}{x} g(x) =$$

$$= \left(\frac{x}{1-x}\right)^{\alpha} \frac{1}{x} g(x)$$

$\left| y = \frac{x}{1-x} \right.$ $\qquad (1-x)y = x \to x = \frac{y}{1+y}$

$$dx = \frac{1}{(1+y)^2} dy$$

$x \to 0 : \quad y \to 0$

$x \to 1^- \quad y \to \infty$

$$\int_0^\infty (y)^\alpha \frac{1+y}{y} g\left(\frac{y}{1+y}\right) \frac{1}{(1+y)^2} dy = \int_0^\infty y^{\alpha-1} \frac{g\left(\frac{y}{1+y}\right)}{1+y} dy$$

$$f(y) = \frac{g\left(\frac{y}{1+y}\right)}{1+y} \qquad = \int_0^\infty y^{\alpha-1} f(y)\,dy$$

$$\boxed{I = \int_{-1}^{1} \frac{(1+x)^{1-p}(1-x)^{p}}{1+x^2}\,dx \qquad -1 < p < 2}$$

$$\frac{(1+x)^{1-p}(1-x)^{p}}{1+x^2} = \Big(\underbrace{\frac{1-x}{1+x}}_{y}\Big)^{p} \quad \frac{1+x}{1+x^2}$$

$$p=0 \quad \int_{-1}^{1} \frac{1}{(1+x)(1+x^2)}\,dx$$