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Linear dynamics of composition operators induced by odometers

Frederic Bayart (University Clermont Auvergne, Clermont-Ferrand, France)

Linear dynamics, namely the study of orbits of linear operators, is a widely studied subject in functional analysis. One is often interested in determining in a special class of operators how to characterize a particular dynamical properties. An important class of operators is that given by composition operators acting on Lebesgue spaces. Among them, the class of composition operators induced by odometers is particularly intriguing because odometers are conservative transformations. We plan in this series of lectures to give an overview of the linear dynamics of composition operators induced by odometers. We will discuss both standard results on linear dynamics and very recent developments.

Dynamical Systems and Descriptive Set Theory

Su Gao (Nankai University, Tianjin, China)

I will give a survey of the descriptive set theoretic framework which are used to measure the relative complexity of classification problems in ergodic theory and topological dynamics. Then I will discuss some concrete classification problems and what is known and not known about their complexity. These include the isomorphism of measure-preserving transformations and the topological conjugacy and the orbit equivalence of Cantor minimal systems.

Zygmund's program and beyond

Laurent Moonens (Paris-Saclay University, Paris, France)

It is well-know that the Hardy-Littlewood (cubical) maximal operator, defined for an integrable function f and $x \in \mathbb{R}^n$ by:

$$Mf(x) := \sup_{x \in Q \text{ cube}} \frac{1}{|Q|} \int_Q |f|,$$

enjoys a weak-type $(1,1)$ inequality (here cubes have all their edges parallel to the coordinate axes). The latter Markov-type inequality on the level sets of Mf allows, for example, to obtain a simple proof of the Lebesgue differentiation theorem, stating that for any $f \in L^1(\mathbb{R}^n)$, averages $\frac{1}{|Q|}\int_Q f$ converge to $f(x)$ when the cube Q shrinks to x, for almost every $x \in \mathbb{R}^n$.

A fascinating program, referred to as Zygmund's program, aims at understanding the optimal weak-type behavior of maximal operators associated in a similar fashion with translation-invariant families $\mathcal{B}$ of n-dimensional parallelepipeds with edges parallel to the coordinate axes:

$$M_{\mathcal{B}} f(x) := \sup_{x \in B \in \mathcal{B}} \frac{1}{|B|} \int_B |f|.$$

When sets in $\mathcal{B}$ may be chosen to have arbitrary small diameters, these optimal weak-type behaviors are also intimately related to understanding the largest space in which a Lebesgue-type differentiation theorem holds with cubes replaced by sets of $\mathcal{B}$. Surprisingly enough, these behaviors can only be completely classified (in terms of geometric overlapping conditions enjoyed by elements of $\mathcal{B}$) in the two-dimensional case, after a work by A. Stokolos.

These series of lectures will aim at presenting the general framework in which Zygmund's program finds its place, and to give a panorama of some known facts, recent works and open problems in (and beyond) Zygmund's program.

Dynamical systems with the specification property

Marcin Sabok (McGill University, Montreal, Canada)

The notion of specification was introduced by Rufus Bowen in the 1970's to study certain types of diffeomorphisms. In this series of lectures we will start with a gentle introduction to the specification property and its connections to other dynamical properties. Then we will focus attention on the possibility of classification of dynamical systems with the specification property and recent results in this direction.