

Monotone assignments in compact and function spaces

A fundamental problem in the study of spaces of continuous functions $C_p(X)$, in the point-wise convergence topology, is to establish the relation between the topological properties of a space X and its function space $C_p(X)$. A natural reason to consider the point-wise topology is that, among all natural topologies for $C(X)$, it is the weaker and hence contains more compact spaces. When X is a compact space, the space $C_p(X)$ has some particularly interesting properties.

First, we will discuss the D -property of spaces $C_p(X)$. This study was initiated by Buzyakova, who obtained some results that generalize simultaneously the classic Theorems of Grothendieck and Baturov about the behaviour of compact like-properties in subspaces of $C_p(X)$. After that, we will present the approach of Tkachuk to improve that results, is here where monotone assignments play a fundamental role. We will present some duality results in C_p -spaces associated with these monotone assignments, which generalize some results obtained by Gruenhague and Tkachuk. We also deal with r -skeletons introduced by Kubiś. An interesting relation, established by Cúth and Kalenda, between monotone assignments and r -skeletons will be revised. Next we will present an study of Corson and Valdivia compact spaces from this point of view. We also will use monotone assignments and r -skeletons to find Corson compact spaces inside the spaces of continuous functions $C_p(X)$. Finally, several open questions in this topic, particularly some related to the classification of compact spaces, will be commented.