(1) $g(u,v) = \sin(u \cdot v)$, kde $u = xy$

$v = x-y$

$h(u(x,y), v(x,y)) = \sin(u(x,y) \cdot v(x,y))$

$$\frac{\partial h}{\partial x} = \frac{\partial g}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial g}{\partial v} \cdot \frac{\partial v}{\partial x}$$

$$= \cos(uv)\, v \cdot y + \cos(uv) \cdot u \cdot 1$$

$$= \cos(xy(x-y))\,(x-y)\,y + \cos(xy(x-y))\, xy$$

Srovnej s:
$$\frac{\partial \sin(xy(x-y))}{\partial x} = \cos(xy(x-y)) \cdot \left(y(x-y) + xy\right)$$

$$\frac{\partial h}{\partial y} = \frac{\partial g}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial g}{\partial v} \cdot \frac{\partial v}{\partial y}$$

$$= \cos(uv)\, v \cdot x + \cos(uv) \cdot u \cdot (-1)$$

$$= \cos(xy(x-y))\,(x-y) \cdot x + \cos(xy(x-y))\, xy\,(-1)$$

Srovnej s
$$\partial \frac{\sin(xy(x-y))}{\partial y} = \cos(xy(x-y)) \cdot \left(x(x-y) + xy(-1)\right)$$

$$F(x,y,z) = x^n f\left(\frac{y}{ax}, \frac{z}{by}\right)$$

$$\overset{?}{} \quad x\frac{\partial F}{\partial x} + y\frac{\partial F}{\partial y} + z\frac{\partial F}{\partial z} = n\cdot F \qquad a,b\in\mathbb{R}$$

$$\frac{1}{a}y\cdot\frac{1}{x}$$

$$f(u,v) \qquad u=\frac{y}{ax} \qquad v=\frac{z}{by}=\frac{1}{b}z\cdot\frac{1}{y}$$

$$F(x,y,z)=x^n f\big(u(y,x),\, v(y,z)\big)$$

$$\frac{\partial F}{\partial x} = nx^{n-1}f(u,v) + x^n\left[\frac{\partial f}{\partial u}\cdot\underbrace{\frac{\partial u}{\partial x}}_{\frac{1}{a}y\cdot\frac{-1}{x^2}} + \frac{\partial f}{\partial v}\cdot\underbrace{\frac{\partial v}{\partial x}}_{0}\right]$$

$$\frac{\partial F}{\partial y} = x^n\left[\frac{\partial f}{\partial u}\cdot\underbrace{\frac{\partial u}{\partial y}}_{\frac{1}{ax}} + \frac{\partial f}{\partial v}\cdot\underbrace{\frac{\partial v}{\partial y}}_{\frac{1}{b}z\cdot -\frac{1}{y^2}}\right]$$

$$\frac{\partial F}{\partial z} = x^n\left[\frac{\partial f}{\partial u}\cdot\underbrace{\frac{\partial u}{\partial z}}_{0} + \frac{\partial f}{\partial v}\cdot\underbrace{\frac{\partial v}{\partial z}}_{\frac{1}{by}}\right]$$

$$x\left(nx^{n-1}f(u,v) + x^n\left[\cancel{\frac{\partial f}{\partial u}\cdot\frac{-y}{ax^2}}\right]\right) +$$

$$y x^n\left[\cancel{\frac{\partial f}{\partial u}\cdot\frac{1}{ax}} + \cancel{\frac{\partial f}{\partial v}\cdot\frac{-z}{by^2}}\right] +$$

$$z x^n\left[\cancel{\frac{\partial f}{\partial v}\cdot\frac{1}{by}}\right] \overset{?}{=} n x^n f(u,v)$$

✓

(2) Nechť $F = (F_1, F_2) : \mathbb{R}^2 \to \mathbb{R}^2$ je zobr. definované

$$F(x,y) = \left(e^{x+y} \sin(x+y),\ \arctan(x^2+y)\right).$$

Zobr. $G = (G_1, G_2) : \mathbb{R}^2 \to \mathbb{R}^2$ splňuje $G(0,0) = (0,0)$

a v $(0,0)$ derivaci repr. maticí

$$\begin{pmatrix} 3 & 2 \\ -2 & -1 \end{pmatrix}$$

(a) ukažte, že v bodě $(0,0)$ $\exists$ dce $F \circ G$ a spočtěte její repr. matici

(b) Spočtěte der. fce $u \mapsto G_1(\sin u, u)$ v bodě 0.

(a) ∂F:

$$\frac{\partial F_1}{\partial x} = e^{x+y} \sin(x+y) + e^{x+y} \cos(x+y), \quad \frac{\partial F_1}{\partial y} = \frac{1}{1}$$

$$\frac{\partial F_2}{\partial x} = \frac{1}{1+(x^2+y)^2} \cdot 2x \qquad \frac{\partial F_2}{\partial y} = \frac{1}{1+(x^2+y)^2}$$

v $(0,0)$ máme

$$\begin{pmatrix} 0+1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

+ parc. dce jsou spojité, dce $\exists$.

$F \circ G$:

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 3 & 2 \\ -2 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -2 & -1 \end{pmatrix}$$

dce složení také $\exists$ (věta)

(b) $$\frac{\partial G_1}{\partial x}(0,0) = 3 \qquad \frac{\partial G_1}{\partial y}(0,0) = 2$$

$h(u) = G_1(\sin u, u) = G_1(x,y)$, kde $x = \sin u$, $y = u$

$$\frac{\partial h}{\partial u} = \frac{\partial G_1}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial G_1}{\partial y} \cdot \frac{\partial y}{\partial u} = \frac{\partial G_1}{\partial x} \cdot \cos u + \frac{\partial G_1}{\partial y} \cdot 1$$

Pro bod $u=0$ $\rightarrow$ $G_1(0,0)$ máme

$$\frac{\partial h}{\partial u}(0) = 3 \cdot \cos 0 + 2 \cdot 1 = \underline{\underline{5}}$$