$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \qquad \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f(x, y) \qquad a = [a_1, a_2]$$

$$\frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(a_1 + h, a_2) - f(a_1, a_2)}{h}$$

$$\frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(a_1, a_2 + h) - f(a_1, a_2)}{h}$$

$|x|$ $\sqrt{x^2}$ $(|x|)' \Rightarrow (\sqrt{x^2})' \Rightarrow$ pr. łańcuchowa $\Rightarrow \operatorname{sgn} x$ $x \neq 0$

2a) $f(x,y) = |x^2 - y^2|$

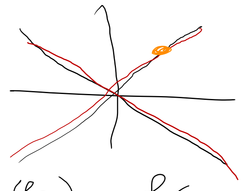
1) $D_f: [x,y] \in \mathbb{R}^2$

2) $\frac{\partial f}{\partial x} = \operatorname{sgn}(x^2-y^2) \cdot 2x$

$\frac{\partial f}{\partial y} = \operatorname{sgn}(x^2-y^2) \cdot (-2y)$

$D_{\frac{\partial f}{\partial x}}, D_{\frac{\partial f}{\partial y}}$: $x^2 - y^2 \neq 0$ $\quad x^2 \neq y^2$

(3) co zdy $x^2 = y^2$?



ustalmy $[x_0, y_0]$: $x_0^2 = y_0^2$

$$\frac{\partial f}{\partial x}[x_0,y_0] = \lim_{h\to 0} \frac{f(x_0+h, y_0) - f(x_0,y_0)}{h}$$

$$= \lim_{h\to 0} \frac{|(x_0+h)^2 - y_0^2| - |x_0^2 - y_0^2|}{h} \quad (0)$$

$$= \lim_{h\to 0} \frac{|x_0^2 + 2x_0h + h^2 - y_0^2|}{h} \quad (0)$$

$$= \lim_{h\to 0} \frac{|2x_0h + h^2|}{h}$$

$$= \lim_{h\to 0} \frac{|h|}{h} |2x_0 + h| \quad (-3 + h)$$

$\lim_{h\to 0+} 1 \cdot |2x_0|$

$\lim_{h\to 0-} -1 \cdot |2x_0|$

$x_0 = 0$ $\lim_{h\to 0} = 0$

$x_0 \neq 0$ $\lim_{h\to 0} \not\exists$

$\frac{\partial f}{\partial x}(x_0,y_0) = \begin{cases} \not\exists \\ 0 \end{cases}$ $x_0 = 0 \ (y_0 = 0)$

$\frac{\partial f}{\partial y}$ wyjdzie stejnie

$\frac{\partial f}{\partial y}(x_0,y_0) = \begin{cases} 0 \\ \not\exists \end{cases}$ $x_0 = 0 = y_0$

inaczej

Tečna v $a = [1,2]$

$$\frac{\partial f}{\partial x}(1,2) = \operatorname{sgn}(1^2-2^2)\cdot 2\cdot 1 = -2$$

$$\frac{\partial f}{\partial y}(1,2) = \operatorname{sgn}(1^2-2^2)\cdot(-2)\cdot 2 = 4$$

$$f(1,2) = |1^2-2^2| = 3$$

Tečna rovina

$$z-3 = -2(x-1)+4(y-2)$$

$$z-3 = -2x+2+4y-8$$

$$z = -2x+4y-3$$

Pozn.

- $|\ | = 0$
- $\sqrt{\ } = 0$ $\qquad \sqrt{x^2}$
- $\arcsin(\ ) = \pm 1$
- f def po částech
- max, min

2c) $\sqrt{y + \sin x}$ $\qquad y = -\sin x$

dce $\qquad y > -\sin x$