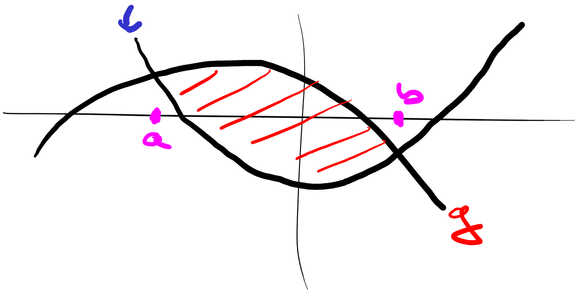
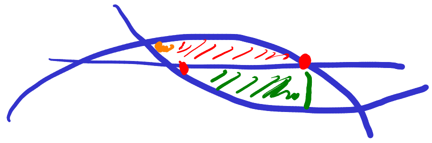
• $f = \underline{x^2 + x - 3}$ $\qquad g(x) = -x^2 - 2x + 2$



f g



$$\int_a^b g - f$$

průsečíky

$$x^2 + x - 3 = -x^2 - 2x + 2$$

$$2x^2 + 3x - 5 = 0$$

$$x_1 = 1 \qquad x_2 = -5/2$$

$$\int_{-5/2}^{1} -x^2 - 2x + 2 - (x^2 + x - 3)\,dx = \int_{-2,5}^{1} -2x^2 - 3x + 5\,dx$$

$$= \left[-\frac{2}{3}x^3 - \frac{3}{2}x^2 + 5x\right]_{-5/2}^{1} = -\frac{2}{3} - \frac{3}{2} + 5 - \left(\frac{250}{54} - \frac{75}{8} - \frac{25}{2}\right)$$

$$= \frac{343}{24}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

$f(x) = \frac{1}{x^2} \geq 0$

spojitá na $[1,\infty)$

nerostoucí ✓

$$\int_1^{\infty} \frac{1}{x^2}\,dx = \left[\frac{-1}{x}\right]_1^{\infty} = 0 - (-1) = \underline{1}$$

konv.

$\sum$ k $\iff \int$ k

Závěr $\sum \frac{1}{n^2}$ k

Objem tělesa

Vzniklé rotací oblasti mezi křivkami $y=x^2$

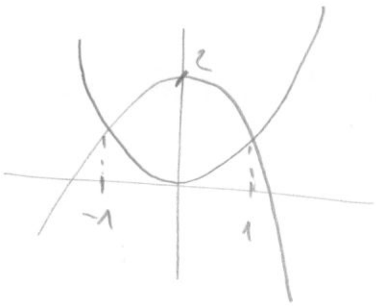
$y=-x^2+2$

kolem osy x

průsečíky: $x^2 = 2-x^2$

$2x^2=2$

$x=\pm 1$



$$V = \pi\int_{-1}^{1} (2-x^2)^2 dx - \pi\int_{-1}^{1} (x^2)^2 dx = \pi\int_{-1}^{1} 4+\cancel{x^4}-4x^2-\cancel{x^4}\, dx$$

$$= \pi\int_{-1}^{1} 4-4x^2 dx = \pi \left[4x-4\frac{x^3}{3}\right]_{-1}^{1} = \pi\cdot 4\left(1-\frac{1}{3}-\left(-1+\frac{1}{3}\right)\right)=$$

$$=\frac{16}{3}\pi$$

Délka asteroidy

$x = \cos^3 t \qquad y = \sin^3 t \qquad t \in [0, 2\pi]$

$x'(t) = 3\cos^2 t(-\sin t)$

$y'(t) = 3\sin^2 t(\cos t)$

$$l = 4\int_0^{\pi/2} \sqrt{(-3\cos^2 t \sin t)^2 + (3\sin^2 t \cos t)^2}\, dt =$$

$$= 4\int_0^{\pi/2} \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t}\, dt =$$

$$= 4\int_0^{\pi/2} \sqrt{9\cos^2 t \sin^2 t\, (\cos^2 t + \sin^2 t)}\, dt = 4\int_0^{\pi/2} 3\, |\cos t \sin t|$$

$$= 12 \int_0^{\pi/2} \cos t \sin t\, dt = 12 \left[\frac{1}{2}\sin^2 t\right]_0^{\pi/2} = \underline{\underline{6}}$$

(subst. $y = \sin t$)