Odmocniny

$z \neq -1$ $z \neq 4$ $\sqrt[3]{x+2} \neq -1$ $\sqrt[3]{x+2} \neq 4$

$x+2 \neq -1$ $x+2 \neq 64$

$x \neq -3$ $x \neq 62$

$(-\infty, -3)$ $(-3, 62)$

$(62, \infty)$

$$\int \frac{1}{\sqrt[3]{(x+2)^2} - 3\sqrt[3]{x+2} - 4}\, dx$$

2 VoS.

$z = \sqrt[3]{x+2}$ $z^3 - 2 = x \leftarrow \varphi(z)$

$3z^2\, dz = dx$ $\varphi' = 3z^2$ $\underline{z \neq 0}$ ✓

$$\int \frac{1}{z^2 - 3z - 4} \cdot 3z^2\, dz = \int 3 - \frac{3/5}{z+1} + \frac{48/5}{z-4}\, dz$$

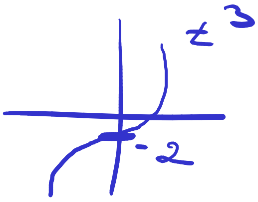
$z \neq -1$ $z \neq 4$

$$\overset{c}{=} 3z - \frac{3}{5}\ln|z+1| + \frac{48}{5}\ln|z-4| =$$

$$= 3\sqrt[3]{x+2} - \frac{3}{5}\ln|\sqrt[3]{x+2}+1| + \frac{48}{5}\ln|\sqrt[3]{x+2}-4|$$

$x \in (a, b)$

$\varphi(z)$ $z \in (\alpha, \beta)$ (a, b)



$(-\infty, -1) \xrightarrow[\text{na}]{\varphi} (-\infty, -3)$

$(-1, 0) \xrightarrow{\varphi} (-3, -2)$

$(0, 4) \longrightarrow (-2, 62)$

$(4, \infty) \longrightarrow (62, \infty)$

v $x = -2$ slepíme

$$\lim_{x \to -2^-} 3\sqrt[3]{x+2} - \frac{3}{5}\ln|\sqrt[3]{x+2}+1| + \frac{48}{5}\ln|\sqrt[3]{x+2}-4| + c_1$$

$$= 0 - 0 + \frac{48}{5}\ln 4 + c_1$$

$$\lim_{x \to -2^+} 3\sqrt[3]{x+2} + \dots + c_2 = \frac{48}{5}\ln 4 + c_2$$

$\Big\}$ $c_1 = c_2$

<u>Závěr</u>

$$G(x) \overset{c}{=} 3\sqrt[3]{x+2} - \frac{3}{5}\ln|\sqrt[3]{x+2}+1| + \frac{48}{5}\ln|\sqrt[3]{x+2}-4|$$

$x \in (-\infty, -3), (-3, 62), (62, \infty)$

1. Vo$

$$dt = \frac{1}{3} \frac{1}{\sqrt[3]{(x+2)^2}} dx$$

$$\int \frac{1}{\sqrt[3]{(x+2)^2} - 3\sqrt[3]{x+2} - 4} \cdot \frac{3\sqrt[3]{(x+2)^2}}{3\sqrt[3]{(x+2)^2}} dx$$

$$\int \frac{3t^2}{3t^2 - 3t - 4} dt$$

$$\int \frac{1}{x^2} \sqrt{\frac{1+x}{x}}\, dx$$

$a_1 \; b_1$
$x \in (-\infty, -1)$
$x \in (0, \infty)$
$a_2 \; b_2$

$$t = \sqrt{\frac{1+x}{x}} \qquad t^2 = \frac{1+x}{x} \qquad x t^2 = 1 + x \qquad x(t^2-1) = 1$$

$$x = \frac{1}{t^2-1}$$

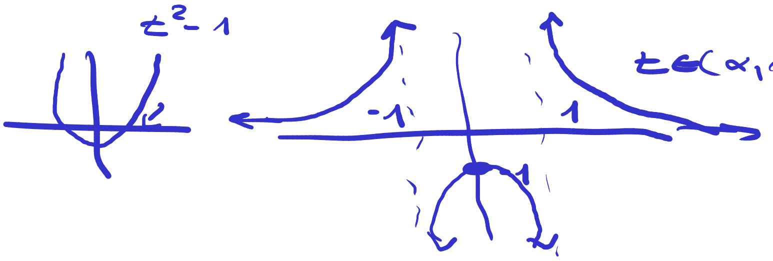
$$dx = -1 \cdot \frac{2t}{(t^2-1)^2}\, dt$$

$$\int \frac{1}{\left(\frac{1}{t^2-1}\right)^2} \cdot t \, \frac{-2t}{(t^2-1)^2}\, dt = \int -2t^2\, dt \overset{c}{=} -\frac{2}{3} t^3$$

$$= -\frac{2}{3} \sqrt{\left(\frac{1+x}{x}\right)^3} \longrightarrow \begin{array}{l} x \in (0, \infty) \\ x \in (-\infty, -1) \end{array}$$

$t \neq \pm 1$

$$\varphi(t) = \frac{1}{t^2-1} \qquad \varphi' = \frac{-2t}{(t^2-1)^2} \qquad t \neq 0$$



$\alpha_1 \; \beta_1 \qquad\qquad a_1 \; b_1$

$(0, 1) \xrightarrow{\varphi \text{ na}} (-\infty, -1)$

$(1, \infty) \xrightarrow{\varphi \text{ na}} (0, \infty)$

$\qquad\qquad\qquad a_2 \; b_2$

Pozn k dalším příkladům

$$\sqrt{(x+2)(x-3)} = \sqrt{\frac{x+2}{x-3} (x-3)^2}$$

$$= \sqrt{\frac{x+2}{x-3}} \cdot |x-3|$$

$$\int \frac{1}{\sqrt{x^2+x+1}}\,dx \qquad x \in \mathbb{R}$$

$$t = \sqrt{x^2+x+1} - x \geq 0$$

$$\geq \sqrt{x^2}$$

$$t + x = \sqrt{x^2+x+1}$$

$$t^2 + 2tx + \cancel{x^2} = \cancel{x^2} + x + 1$$

$$x(2t-1) = 1 - t^2$$

$$x = \frac{1-t^2}{2t-1}$$

$$dx = -2\frac{t^2-t+1}{(1-2t)^2}\,dt$$

$$\int \frac{1}{t + \frac{1-t^2}{2t-1}} \cdot (-2)\frac{t^2-t+1}{(1-2t)^2}\,dt = \int \frac{-2(\cancel{t^2-t+1})}{\frac{\cancel{2t^2-t+1-t^2}}{2t-1}} \cdot \frac{1}{(1-2t)^2}\,dt$$

$$t^2 - t + 1$$

$$= \int -2\,\frac{-1}{1-2t}\,dt \overset{c}{=} \frac{2}{-2}\ln|1-2t| =$$

$$= -\ln\left|1 - 2\left(\sqrt{x^2+x+1} - x\right)\right| \qquad x \in (-\infty, \infty)$$

$$\varphi(t) = \frac{1-t^2}{2t-1} \qquad \varphi'(t) = -2\,\frac{t^2-t+1}{(1-2t)^2} \neq 0 \text{ vždy}$$

$$t \neq \tfrac{1}{2}$$

(α, β)

α, β		(a, b)
~~$(-\infty, 1/2)$~~	$\longrightarrow$	$(\cancel{-\infty}, \infty)$
$(1/2, \infty)$	$\longrightarrow$	$(-\infty, \infty)$
α, β