$$\int \frac{1}{1+\sqrt{x}}\,dx$$

1. Vos

$$\int \frac{1}{1+\sqrt{x}} \cdot \frac{2\sqrt{x}}{2\sqrt{x}}\,dx = \int \frac{1}{1+y}\,2y\,dy = 2\int \frac{y+1-1}{y+1}\,dy$$

$y = \sqrt{x}$

$dy = \frac{1}{2\sqrt{x}}\,dx$

$$= 2\int 1 - \frac{1}{y+1}\,dy \overset{c}{=} 2y - 2\log|1+y|$$

$$= 2\sqrt{x} - 2\log(1+\sqrt{x})$$

$x \in (0,\infty)$
α, β

$\varphi(x) = \sqrt{x}$

$\varphi' = \frac{1}{2\sqrt{x}}$

$\varphi(0,\infty) = (0,\infty) \subseteq (-1,\infty)$

$y \in (-1,\infty)$
(a,b)

$f(y) = \frac{2y}{1+y}$ $\quad F(y) = 2y - 2\log|1+y|$

2. Vos

$$\int \frac{1}{1+\sqrt{x}}\,dx = \int \frac{1}{1+t}\,2t\,dt \overset{c}{=} 2t - 2\log|1+t|$$

$x = t^2 \quad (t = \sqrt{x})$

$$= 2\sqrt{x} - 2\log(1+\sqrt{x})$$

$dx = 2t\,dt$

$f(x) = \frac{1}{1+\sqrt{x}}$

$$\int f(\varphi(t))\cdot\varphi'(t)\,dt = \int \frac{1}{1+\sqrt{t^2}}\,2t\,dt \overset{c}{=} \underbrace{2t - 2\log|1+t|}_{G(t)}$$

$\varphi(t) = t^2$

$\varphi'(t) = 2t$

$\varphi^{-1} = \sqrt{x}$

$t \in (\alpha,\beta) = (0,\infty)$

$\varphi(0,\infty) = (0,\infty) = (a,b)$
$x \in (0,\infty)$

$\varphi' \neq 0$ na $(0,\infty)$

$G(\varphi^{-1}(x)) = 2\sqrt{x} - 2\log(1+\sqrt{x})$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{1-\sin^2 t}}\cos t\,dt = \int \frac{\cos t}{\sqrt{\cos^2 t}}\,dt$$

$\sin t = x$

$\cos t\,dt = dx$

$\arcsin x = t$

$$= \int \frac{\cos t}{|\cos t|} \overset{!}{=} \int \frac{\cos t}{\cos t}\,dt = \int 1\,dt$$

$$\overset{c}{=} t = \arcsin x$$

ZVoS

$\varphi(t) = \sin t$

$\varphi'(t) = \cos t$

$\varphi^{-1}(x) = \arcsin x$

$$f(x) = \frac{1}{\sqrt{1-x^2}}$$

$$f(\varphi(t))\,\varphi'(t) = \frac{1}{\sqrt{1-\sin^2 t}}\cos t$$

$\overset{(\alpha,\beta)}{t \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)}$

$x \in (-1,1) = (a,b)$

$G = t$

$G(\varphi^{-1}(x)) = \arcsin x$

$$\underset{(\alpha,\beta)}{\varphi\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)} = \underset{(a,b)}{(-1,1)}$$

$\varphi' = \cos t \neq 0$ na (α,β)

$$\int \overbrace{\frac{\sqrt{9-x^2}}{x^2}}^{f(x)}\,dx \qquad f = \frac{\sqrt{9-x^2}}{x^2} \quad = (0,3) \quad (-3,0)$$

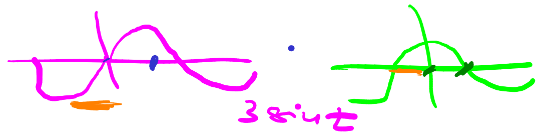
$$(a,b) \quad (a_1, b_1) \quad (\alpha_1, \beta_1) = \left(-\tfrac{\pi}{2}, 0\right)$$

$x = 3\sin t$ $\qquad \varphi(t) = 3\sin t \qquad (\alpha, \beta) = \left(0, \frac{\pi}{2}\right)$

$dx = 3\cos t\, dt$ $\qquad \varphi'(t) = 3\cos t \qquad \varphi\left(0, \frac{\pi}{2}\right) = (0,3)$

$t = \arcsin \frac{x}{3}$ $\qquad \varphi^{-1}(x) = \arcsin \frac{x}{3} \qquad 3\cos t \neq 0$ na $\left(0, \frac{\pi}{2}\right)$

$$\underbrace{\int \frac{\sqrt{9 - 9\sin^2 t}}{9\sin^2 t}\, 3\cos t}_{f(\varphi)\cdot\varphi'}\, dt =$$

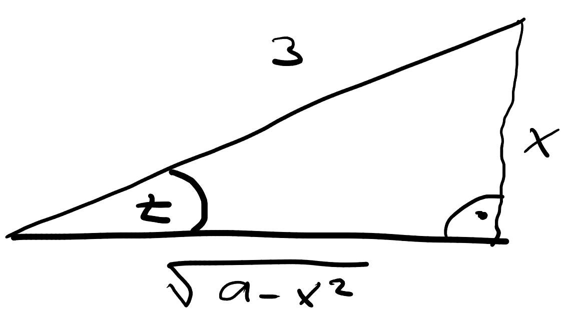


$$= \int \frac{\sqrt{9}\sqrt{1-\sin^2 t}}{3\sin^2 t}\cos t\, dt = \int \frac{\sqrt{\cos^2 t}}{\sin^2 t}\cos t\, dt$$

$$\overset{!}{=} \int \frac{\cos^2 t}{\sin^2 t}\, dt = \int \frac{1-\sin^2 t}{\sin^2 t}\, dt = \int \frac{1}{\sin^2 t} - 1\, dt$$

$$\overset{c}{=} \underbrace{-\cot g\, t - t}_{G(t)} = \underbrace{-\cot g\left(\arcsin \frac{x}{3}\right) - \arcsin \frac{x}{3}}_{G(\varphi^{-1})}$$

$$= -\frac{\sqrt{9-x^2}}{x} - \arcsin \frac{x}{3}$$



$$\frac{x}{3} = \sin t$$

$$\cot t = \frac{\sqrt{9-x^2}}{x}$$