Lepení

$$f(x) = \begin{cases} 3x^2 & x > 0 \\ 0 & x = 0 \\ -3x^2 & x < 0 \end{cases}$$

PF k f ?

Hledáme F(x)

- f spojitá na $\mathbb{R}$ $\longrightarrow$ F bude na $\mathbb{R}$

- $$F(x) = \begin{cases} x^3 + c & x \in (0, \infty) \\ -x^3 + d & x \in (-\infty, 0) \end{cases}$$

- Řešíme $x = 0$

$$\lim_{x \to 0-} F(x) = \lim_{x \to 0+} F(x)$$

$$\lim_{x \to 0-} -x^3 + d = \lim_{x \to 0+} x^3 + c$$

$$\boxed{d = c}$$

Závěr

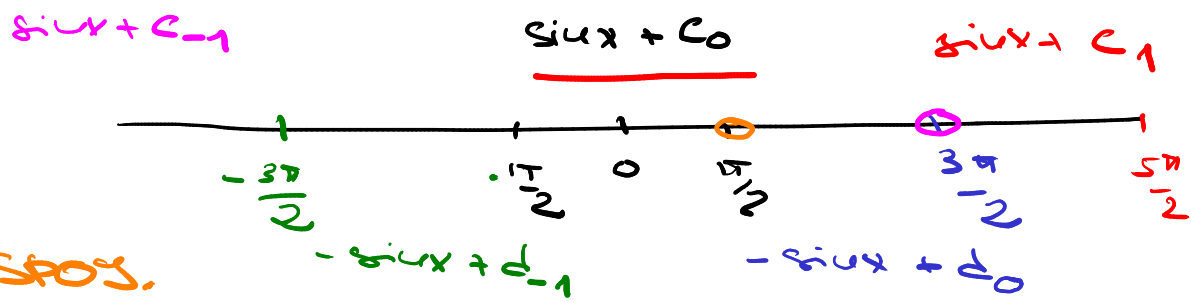
- $$F(x) = \begin{cases} x^3 + c & x > 0 \\ 0 + c & x = 0 \\ -x^3 + c & x < 0 \end{cases}$$

$f = |\cos x|$

$$f = \begin{cases} \cos x & -\frac{\pi}{2} < x < \frac{\pi}{2} \quad +2k\pi \\ 0 & \frac{\pi}{2} + k\pi \\ -\cos x & \frac{\pi}{2} < x < \frac{3\pi}{2} \quad +2k\pi \end{cases}$$

- f spoj. na $\mathbb{R}$

$$F = \begin{cases} \sin x + c_k, & x \in (-\frac{\pi}{2}, \frac{\pi}{2}) + 2k\pi \\ & \sin x + c_0 + 4k \\ & c_0 + 1 + 2k \quad x = \frac{\pi}{2} + 2k\pi \\ -\sin x + d_k; & -\sin x + c_0 + 2 + 4k \\ & x \in (\frac{\pi}{2}, \frac{3\pi}{2}) + 2k\pi \end{cases}$$



CHCEME SPOJ.

$\frac{\pi}{2}$:

$$\lim_{x \to \frac{\pi}{2}-} F(x) = \lim_{x \to \frac{\pi}{2}+} F(x)$$

$$\lim_{x \to \frac{\pi}{2}-} \sin x + c_0 = \lim_{x \to \frac{\pi}{2}+} -\sin x + d_0$$

$$1 + c_0 = -1 + d_0$$

$$\boxed{c_0 + 2 = d_0}$$

$\frac{3\pi}{2}$

$$\lim_{x \to \frac{3\pi}{2}-} -\sin x + d_0 = \lim_{x \to \frac{3\pi}{2}+} \sin x + c_1$$

$$+1+d_0 = -1+c_1$$

$$+2+d_0 = c_1$$

$$c_0+2+2 = c_1$$

$$c_0+4 = c_1$$

Dohromady vzjde

$$c_k = c_0+4k$$

$$d_k = c_0+2+4k$$

$$\frac{\pi}{2} = c_0+1 \qquad \frac{\pi}{2}+2k\pi : \; c_0+1+2k$$

$$\frac{3\pi}{2} = c_0+3$$

Obecně

$$\lim_{x\to(\frac{\pi}{2}+2k\pi)-} F(x) = \lim_{x\to(\frac{\pi}{2}+2k\pi)+} F(x)$$

$$\lim_{x\to(\frac{\pi}{2}+2k\pi)-} \sin x + c_k = \lim_{x\to(\frac{\pi}{2}+2k\pi)+} -\sin x + d_k$$

$$1+c_k = -1+d_k$$

$$c_k+2 = d_k$$

$$\lim_{x\to(\frac{3}{2}+2k\pi)-} F(x) = \lim_{x\to(\frac{3\pi}{2}+2k\pi)+}$$

$$\lim_{x\to(\frac{3}{2}\pi+2k\pi)-} -\sin x + d_k = \lim_{x\to(\frac{3\pi}{2}+2k\pi)+} \sin x + c_{k+1}$$

$$-1+d_k = 1+c_{k+1}$$

$$d_k+2 = c_{k+1}$$