Substituce

Př $\int \underbrace{\cos(x^2)\cdot 2x}_{\text{spoj na } \mathbb{R}}\, dx = \int \cos y\, dy \overset{c}{=} \sin(y) = \sin(x^2) \qquad x \in \mathbb{R}$

$y = x^2$

$dy = 2x\,dx$

$\varphi(x) = x^2 \qquad f(y) = \cos y$

$\varphi'(x) = 2x \qquad F(y) = \sin y$

$\varphi : (\alpha, \beta) \to (a, b) \qquad f : (a,b) \to \mathbb{R}$

$\mathbb{R} \to [0, \infty) \subset \mathbb{R} = (a,b) \qquad \mathbb{R} \to \mathbb{R}$

Př $\int \underbrace{\frac{1}{\sin x} \cdot \cos x}_{\text{spoj na } (0,\pi) + k\pi}\, dx = \int \frac{1}{y}\, dy \overset{c}{=} \log|y| = \log|\sin x|$

$x \in (0, \pi) + k\pi$

$k \in \mathbb{Z}$

$y = \sin x$

$dy = \cos x\, dx$

$\varphi(x) = \sin x \qquad f(y) = \frac{1}{y}$

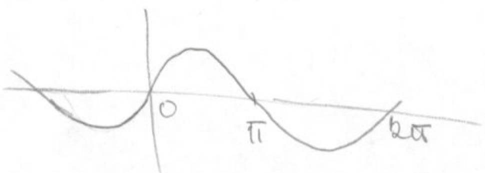
$\varphi'(x) = \cos x \qquad F(y) = \log|y|$

$\varphi : (\alpha, \beta) \to (a, b) \qquad f : (a,b) \to \mathbb{R}$

$\underset{+2k\pi}{(0, \pi)} \to (0, 1] \qquad (0, \infty)$

$(\bar\alpha, \bar\beta) \to (\bar a, \bar b) \qquad (\bar a, \bar b) \to \mathbb{R}$

$\underset{+2k\pi}{(\pi, 2\pi)} \to [-1, 0) \qquad (-\infty, 0) \to \mathbb{R}$



Per partes

Př. $\int x \sin x \, dx = x(-\cos x) - \int -\cos x \, dx$

$u = x \quad v' = \sin x$ $\quad \stackrel{c}{=} -x\cos x + \sin x \quad x \in \mathbb{R}$

$u' = 1 \quad v = -\cos x$

Př. $\int 1 \cdot \arctan x \, dx = x \arctan x - \int \frac{x}{1+x^2} \, dx$

$v' = 1 \quad u = \arctan x$ $\quad = x \arctan x - \frac{1}{2} \int \frac{2x}{1+x^2} \, dx$

$v = x \quad u' = \frac{1}{1+x^2}$

$\stackrel{c}{=} x \arctan x - \frac{1}{2} \log|1+x^2| \quad x \in \mathbb{R}$

Př. $\int e^x \cos x \, dx = e^x \sin x - \int e^x \sin x \, dx$

$u = e^x \quad v' = \sin x$

$u' = e^x \quad v = -\cos x$

$u = e^x \quad v' = \cos x$

$u' = e^x \quad v = \sin x$

$= e^x \sin x - (-e^x \cos x + \int e^x \cos x)$

$= e^x \sin x + e^x \cos x - \int e^x \cos x$

$2 \int e^x \cos x \stackrel{c}{=} e^x \sin x + e^x \cos x$

$\int e^x \cos x \stackrel{c}{=} \frac{1}{2}(e^x \sin x + e^x \cos x)$