

① $\lim_{n \rightarrow \infty} \frac{n! + \sqrt{n} + \log n^4 - \frac{5}{n^4}}{\sqrt[4]{n^3} + 3 \log n + 6 \cos n - 2n!} =$

$= \lim_{n \rightarrow \infty} \frac{\cancel{n!}}{\cancel{n!}} \cdot \frac{1 + \frac{\sqrt{n}}{n!} + 4 \frac{\log n}{n!} - \frac{5}{n^4 \cdot n!}}{\frac{\sqrt[4]{n^3}}{n!} + 3 \frac{\log n}{n!} + 6 \frac{\cos n}{n!} - 2} \stackrel{\text{AL}}{=} \frac{1+0+0-0}{0+0+0-2} = \frac{1}{-2} = -\frac{1}{2}$

0 stałe *meżna a więcej!*

② $\lim_{n \rightarrow \infty} \sqrt{4n} (\sqrt{n^3-1} - \sqrt{n^3-n})$

$= \lim_{n \rightarrow \infty} \sqrt{4n} \frac{(n^3-1) - (n^3-n)}{\sqrt{n^3-1} + \sqrt{n^3-n}} =$

$= \lim_{n \rightarrow \infty} \frac{(n-1) 2\sqrt{n}}{\sqrt{n^3-1} + \sqrt{n^3-n}} = \lim_{n \rightarrow \infty} \frac{2n\sqrt{n}}{n\sqrt{n}} \cdot \frac{1 - \frac{1}{n}}{\sqrt{1 - \frac{1}{n^3}} + \sqrt{1 - \frac{1}{n^2}}}$

$\stackrel{\text{AL}}{=} 2 \cdot \frac{1-0}{\sqrt{1-0} + \sqrt{1-0}} = 1$

Vo V

③ $\lim_{n \rightarrow \infty} \sqrt[n]{\frac{1}{2^n} + n + 4^n + \sqrt{n}} = 4$

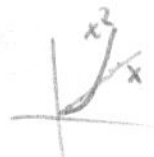
$4 = \sqrt[n]{0+0+4^n+0} \leq \sqrt[n]{\frac{1}{2^n} + n + 4^n + \sqrt{n}} \leq \sqrt[n]{4 \cdot 4^n} = 4 \sqrt[n]{4}$

\downarrow \downarrow 2 policajki \downarrow 4.1

4 4 4.1

$\frac{1}{2^n} \leq 4^n$ $\sqrt{n} \leq n \leq 4^n$ $\lim_{n \rightarrow \infty} \frac{n}{4^n} = 0$

$1 \leq 2^4 4^n$ \downarrow $\text{płaci } \forall n \geq 1$ $\text{tedy } \exists n_0: \forall n \geq n_0 \frac{n}{4^n} \leq 1$

$\geq \text{Fejma}$ 

(4) (a) Jest liže \exists vl. $\lim [a_n]$ \Rightarrow \exists vl. $\lim a_n$

heptavala, protipriklad

$$a_n = \begin{cases} 0 & n \text{ sudé} \\ \frac{1}{2} & n \text{ liché} \end{cases} \quad \text{paž } [a_n] = 0$$

$\lim [a_n] = 0$, ale $\lim a_n \nexists$

(b) Jest liže \exists vl. $\lim a_n$ \Rightarrow \exists vl. $\lim [a_n]$

heptavala, protipriklad

$$a_n = \left(-\frac{1}{2}\right)^n \quad \begin{array}{c} | \cdot \cdot \cdot \cdot \cdot \\ \cdot \cdot \cdot \cdot \cdot \end{array} \quad \lim a_n = 0$$

$$\text{paž } [a_n] = \begin{cases} 0 & n \text{ sudé} \\ -1 & n \text{ liché} \end{cases} \quad \lim [a_n] \nexists$$