(1) $\lim \int_0^1 \underbrace{\frac{n e^{-n\sqrt{x}}}{1+x^2}}_{f_n} dx = 0$

1 • fix x: $\lim_{n\to\infty} \frac{n e^{-n\sqrt{x}}}{1+x^2} = 0$ ← stačila

f_n spoj → měřitelné na $(0,1)$

další kandidáti: $g(1,x) = \frac{1}{e^{\sqrt{x}}(1+x^2)}$

$g(\infty,x) = \lim_{n\to\infty} g(n,x) = 0$

3 • $g(n,x) = \frac{n e^{-n\sqrt{x}}}{1+x^2}$

$\frac{\partial g}{\partial n} = \frac{e^{-n\sqrt{x}} + n e^{-n\sqrt{x}}(-\sqrt{x})}{1+x^2} \Rightarrow e^{-n\sqrt{x}}(1-n\sqrt{x}) = 0$

$n_0 = \frac{1}{\sqrt{x}}$

$g(n_0,x) = \frac{\frac{1}{\sqrt{x}} e^{-\frac{1}{\sqrt{x}}\sqrt{x}}}{1+x^2} = \frac{1/e}{\sqrt{x}(1+x^2)}$

$g(n_0,x)$ u 1 spoj

u 0 $\approx \frac{1}{\sqrt{x}}$

1 $f_n \le g(n_0,x) = \frac{1}{e\sqrt{x}(1+x^2)}$ ← majoranta

1 Tedy $\lim \int = \int \lim = \int_0^1 0 = 0.$

(2) $\int_0^1 \cos x \ln x \, dx$

f_n spoj → měř. na $(0,1)$

1 $\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ pro $x \in \mathbb{R}$ $\Rightarrow \int_0^1 \sum_{n=0}^{\infty} \overbrace{(-1)^n \frac{x^{2n}}{(2n)!} \ln x}^{f_n} \, dx$

3 $\sum_{n=0}^{\infty} \int_0^1 \left| (-1)^n \frac{x^{2n}}{(2n)!} \ln x \right| dx = \sum_{n=0}^{\infty} \frac{-1}{(2n)!} \int_0^1 x^{2n} \ln x \, dx = \sum_{n=0}^{\infty} \frac{1}{(2n)!(2n+1)^2}$ K $\left(< \sum \frac{1}{n^2}\right)$

$\int_0^1 x^{2n} \ln x \, dx = \left[\frac{x^{2n+1}}{2n+1} \ln x\right]_0^1 - \int_0^1 \frac{x^{2n}}{2n+1} dx = -\left[\frac{x^{2n+1}}{(2n+1)^2}\right]_0^1 = \frac{-1}{(2n+1)^2}$

u' v

$u = \frac{x^{2n+1}}{2n+1}$ $v' = 1/x$

Lebesgue →

2 $\int_0^1 \cos x \ln x \, dx = \sum_{n=0}^{\infty} \int_0^1 \frac{(-1)^n x^{2n}}{(2n)!} \ln x = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{(2n)!(2n+1)^2}$