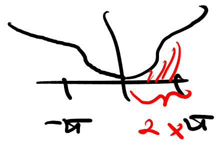
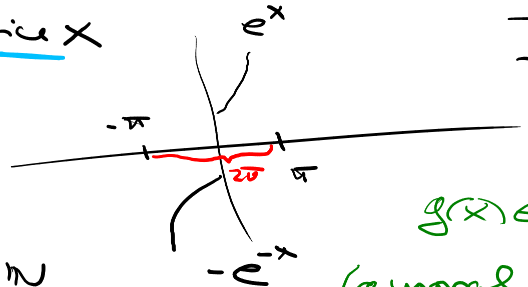
$f(x)=e^x \qquad [0,\pi)$

↳ FŘ obsahuje pouze $\sin x$





- liché kopie $g(x)$

$g(x)\in BV([0,2\pi])$

(g monot. na $(0,\pi)$, $(\pi,2\pi)$, ...)

- koef. $a_0, a_n = 0 \qquad n\in\mathbb{N}$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx) = \frac{2}{\pi}\int_0^{\pi} f(x)\sin(nx)\,dx$$

$n\in\mathbb{N}$

$$= \frac{2}{\pi}\int_0^{\pi} e^x \sin(nx)\,dx = \frac{2}{\pi}\left[\frac{e^x}{1+n^2}(\sin nx - n\cos nx)\right]_0^{\pi}$$

Jinak :)

$$e^{a+bi} = e^a(\cos b + i\sin b)$$

$\sin(nx) = \operatorname{Im} e^{inx}$

imag. složka

$$\Rightarrow = \frac{2}{\pi(1+n^2)}(-e^{\pi} n\cos n\pi + n)$$

$$\int e^x \sin(nx)\,dx = \int e^x \operatorname{Im}(e^{inx})\,dx = \operatorname{Im}\int e^x e^{inx}\,dx$$

$$\operatorname{Im}\int e^{x(1+in)}\,dx = \operatorname{Im}\frac{e^{x(1+in)}}{1+in}\cdot\frac{1-in}{1-in} =$$

$$\operatorname{Im}\frac{1-in}{1-(i^2n^2)}e^x(\cos(nx)+i\sin(nx)) =$$

$$\operatorname{Im}\frac{e^x}{1+n^2}\left(\cos nx + \underline{i\sin nx} - \underline{in\cos nx} - in\, i\sin nx\right)$$

$$= \frac{e^x}{1+n^2}(\sin(nx) - n\cos(nx))$$

$$b_n = \frac{2}{\pi(1+n^2)}(n - e^{\pi} n(-1)^n)$$

$$F = \sum_{n=1}^{\infty} \frac{2}{\pi(1+n^2)}(n - e^{\pi} n(-1)^n)\sin(nx)$$