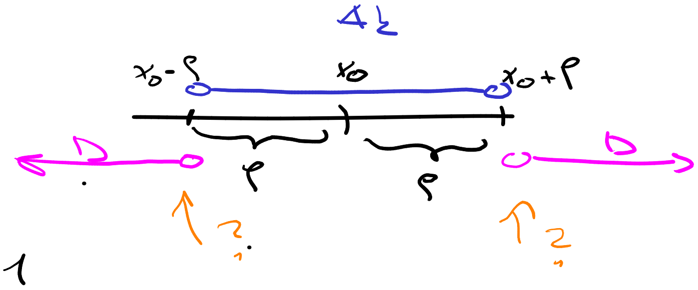
Mocninné řady

x_0 – střed m. ř.

$$\sum_{n=0}^{\infty} \underbrace{a_n (x-x_0)^n}_{f_n(x)}$$

ρ $\qquad \sum$ AK $\quad : |x-x_0| < \rho$

$\qquad \sum$ D $\quad : |x-x_0| > \rho$



$$\rho = \frac{1}{\limsup\limits_{n\to\infty} \sqrt[n]{|a_n|}}$$

$$\sum_{n=1}^{\infty} \frac{1}{n} x^n$$

- $a_n = \frac{1}{n} \qquad x_0 = 0$
- $\frac{1}{\rho} = \limsup\limits_{n\to\infty} \sqrt[n]{\left|\frac{1}{n}\right|} = \lim\limits_{n\to\infty} \frac{1}{\sqrt[n]{n}} = 1$

$\rho = 1$

- Víme:

$$\begin{array}{ccc} -1 & 0 & 1 \end{array}$$

$x \in (-1,1)$ AK $\qquad x \in (-\infty,-1)$ D

$\qquad\qquad\qquad\qquad (1,\infty)$ D

- $x = 1 \qquad x = -1 \qquad ?$

$\sum \frac{1}{n} 1^n$ D $\qquad \sum \frac{1}{n}(-1)^n$ NAK

Závěr: $\quad x \in (-1,1)$ AK $\qquad x = -1$ NAK

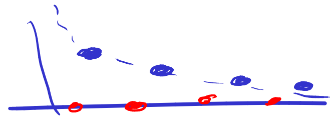
$x \in (-\infty,-1)$ D

$x \in [1,\infty)$ D

$$\sum_{n=1}^{\infty} \frac{1+(-1)^n}{n} x^n$$

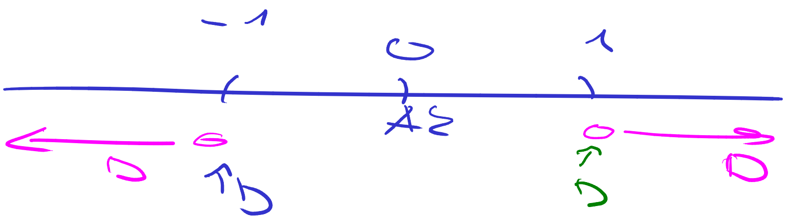
n sudé $\quad \frac{1+1}{n} = \frac{2}{n}$

n liché $\quad \frac{0}{n}$



$$\rho = \frac{1}{\limsup \sqrt[n]{\frac{1+(-1)^n}{n}}}$$

$$= \frac{1}{\lim\limits_{n\to\infty} \sqrt[n]{\frac{2}{n}}} = \frac{1}{\lim \frac{\sqrt[n]{2}}{\sqrt[n]{n}}} = \underline{\underline{1}}$$



• $x = 1$

$$\sum \frac{1+(-1)^n}{n} \quad D$$

$$\underbrace{\sum \frac{1}{n} + \sum \frac{(-1)^n}{n}}_{D}$$

D $\qquad$ L ← Leibniz

$x = -1$

$$\sum \frac{1+(-1)^n}{n} (-1)^n$$

$$\underbrace{\sum \frac{(-1)^n}{n} + \frac{1}{n}}_{D}$$

Závěr: AK $x \in (-1, 1)$

D $x \in (-\infty, -1], [1, \infty)$