Posl. funkcí

$$f_n(x) = \arctan(nx)$$

- bodová limita

fix x

$$\lim_{n\to\infty} \arctan(nx) = \begin{cases} \frac{\pi}{2} & x>0 \\ 0 & x=0 \\ -\frac{\pi}{2} & x<0 \end{cases}$$

$$f(x) = \begin{cases} \pi/2 & x>0 \\ 0 & x=0 \\ -\pi/2 & x<0 \end{cases}$$

- stejn. konv.

fix n

$$\sigma_n = \sup\{|f_n(x) - f(x)|;\ x\in\mathbb{R}\}$$

$$\left|\arctan(nx) - \frac{\pi}{2}\right| \quad x>0$$

$$\left|\arctan(nx) + \frac{\pi}{2}\right| \quad x<0$$

$0 - 0$

$$\sigma_n = \frac{\pi}{2} \qquad \lim_{n\to\infty} \frac{\pi}{2} = \frac{\pi}{2} \neq 0 \quad :($$

$f_n \not\rightrightarrows f$ na $\mathbb{R}$.

Pzn f_n spojí. na $\mathbb{R}$

f není

$\Big\}$ $f_n \not\rightrightarrows f$ na $\mathbb{R}$

$$f_n(x) = \frac{nx}{1+n^2x^2} \qquad D_{f_n} = \mathbb{R}$$

- fix $x \in \mathbb{R} \setminus \{0\}$

$$\lim_{n\to\infty} \underbrace{\frac{n}{n^2}}_{\to 0} \cdot \frac{x}{\underbrace{\frac{1}{n^2}+x^2}_{\frac{1}{x}}} = 0$$

$x=0$ $\quad f_n(0) = \frac{0}{1} \quad \lim_{n\to\infty} = 0$

$$\boxed{f(x) = 0}$$

- fix $\underline{n}$

$$\sup\left\{ \left| \frac{nx}{1+n^2x^2} - 0 \right|, x \in \mathbb{R} \right\}$$

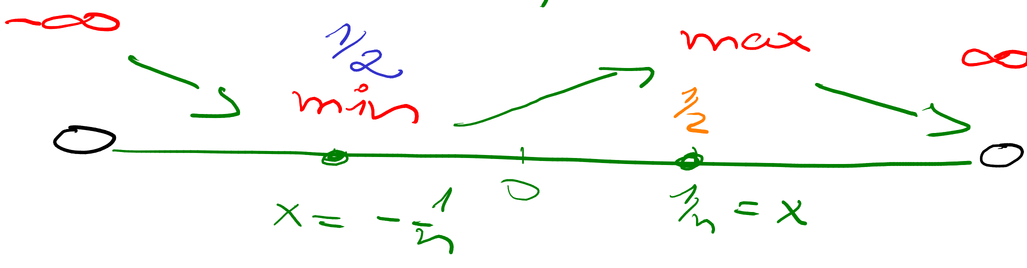
$$g_n(x) = \frac{nx}{1+n^2x^2}$$

$$g_n'(x) = \frac{n - n^3x^2}{(1+n^2x^2)^2}$$

$n = n^3x^2$

$\frac{1}{n^2} = x^2$

$\pm\frac{1}{n} = x$



$$\left|f_n\left(\frac{1}{n}\right)\right| = \left|\frac{n\cdot\frac{1}{n}}{1+n^2\cdot\frac{1}{n^2}}\right| = \left|\frac{1}{2}\right|$$

$$f_n\left(-\frac{1}{n}\right) = \frac{-n\,\frac{1}{n}}{1+n^2\cdot\frac{1}{n^2}} = -\frac{1}{2}$$

$$\lim_{x\to\pm\infty} \frac{nx}{1+n^2x^2} = 0$$

$$\boxed{\sigma_n = \frac{1}{2}} \quad \cdot \quad \lim_{n\to\infty} \frac{1}{2} = \frac{1}{2} \neq 0$$

$$\to \quad f_n \not\rightrightarrows 0$$

- lok. st. konv. ?

$\boxed{x_0 = 0}$ na libo. okolí $(-r, r)$

$$\sup\{|f_n(x) - 0|,\ x \in (-r,r)\}$$

dost velká n $\quad = \frac{1}{2}$

na $(-r,r)$ $\quad f_n \not\rightrightarrows 0$

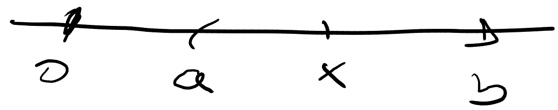
- Závěr: $f_n \not\rightrightarrows f$ na $\mathbb{R}$

$$f_n \overset{\text{lok}}{\not\rightrightarrows} f \quad \text{na } \mathbb{R}$$

$\ulcorner$ $f_n \rightrightarrows f$ na $(0,\infty)$? ✓

Pro $x \in (0,\infty)$ hledejme okolí:

$$0 < a \leq x < b$$



sup teď bude v a :)

$$f_n(a) = \underbrace{\frac{na}{1+n^2a^2}}_{\approx \frac{1}{n}} \xrightarrow{n\to\infty} 0 \quad \checkmark$$

$f_n \rightrightarrows$ na (a,b) :)

Dohromady

$$f_n \overset{loc}{\rightrightarrows} f \quad \text{na } (0,\infty)$$